

Influence of First Shape Factor in Behaviour of Rubber Bearings Base Isolated Buildings.

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Abstract—Rubber bearings isolators are widely used in base isolation system for different kind of structures. Their characteristics depend on components properties (rubber and shims), displacement magnitude, and first and second shape factor. For a given materials, the second shape factor which gave the ratio of isolator diameter to total rubber height, doesn't vary too much. In opposite, the first shape factor given as $S_1 = D / 4t$ (where D and t denotes the isolator diameter and single rubber layer thickness), have a wide range and influence the isolator properties by its quadratic value. By increasing this factor we get a very high modulus of elasticity for the combined materials, which yield in very high vertical stiffness and critical axial load for the isolator unit. The horizontal stiffness is sensitive to ratio of design axial load to critical load and decrease rapidly when this ratio increases. From the other side the critical load depend on shear strain. The reduction in horizontal stiffness yield in higher shear strain and a further reduction in critical load and so on. In this study will be shown from quantitative side of view the influence of first shape factor to the above properties and performance of isolation system under strong motion earthquake. The fast nonlinear and direct integration analyses will be used to observe the performance of a base isolated building with different first shape factors of isolators.

Keywords-isolation system, shape factor, strong motion, critical load, reduced stiffness

I. INTRODUCTION

In this study will be show the behaviour of base isolated building, with laminated rubber bearings, for different values of first shape factor. For this reason a seven storey regular in-plane building is chose with a frame structure. The floor weight is assumed equal to 12 kN/m², which is rather big, just to amplify the response of the structure. A 5% accidental in-plane eccentricity is taken according to Eurocode-8 provisions. The building is subjected to a strong motion earthquake presented by ground accelerogram acting in one direction. Attempting a target effective period of 2.5 seconds and 1.5 desired shear strain, the isolators units with specific second shape factors are chose to support the structures under each column. Then the isolators are taken from no lamination to highly laminate. The superstructure is simplified by modelling each storey with linear link element, assuming that the structures will remain in linear elastic range. The link element have three degree of freedom, horizontal stiffness in "X" and "Y" direction and rotational stiffness in "Z" direction. Modelling of superstructure with link elements reduce drastically the amount of computational calculation and adequately present the whole structure. As concern to rubber isolators, they are modelled with nonlinear link element with two degree of freedom. During the analyses the axial force acting on isolators are invariant and the effective horizontal stiffness are reduced by reduction of critical stresses for different level of shear strains. A variation of axial loads can be included as well with a satisfactory approximation. The performance of base isolated building is carry out by nonlinear time history analyses with time step of 0.02 seconds which is a satisfy threshold to catch the response of every single isolator unit. At the end, based on the analyses results, the proper conclusions are given.

II. BUILDING PARAMETERS AND MODELING

The building consists of seven rectangular storeys with equal height of 3.2m as shown in figure 1. According to Eurocode-8 provisions, the building is classified regular in-plane and a 5% accidental eccentricity is assume. The weight for each floor is taken uniformly distributed with intensity of 12kN/m². Mass properties are attached to a special join with three degree of freedom; mass in "X", "Y" and rotational about "Z" axis. The possible position of the mass centre is given by the hatch area. Because the structure is symmetric, the initial centre of rigidity corresponds with the geometric centre of the building plan. As we will see, this point is not fixed for the isolation system for different level of deformation. For each isolator unit the initial axial static loads are calculated based on respectively supported area. Based on preliminary design, the external columns are taken with square cross-section 50x50cm and inner columns 60x60cm. The design of superstructure is not subject of this study [2] [4].

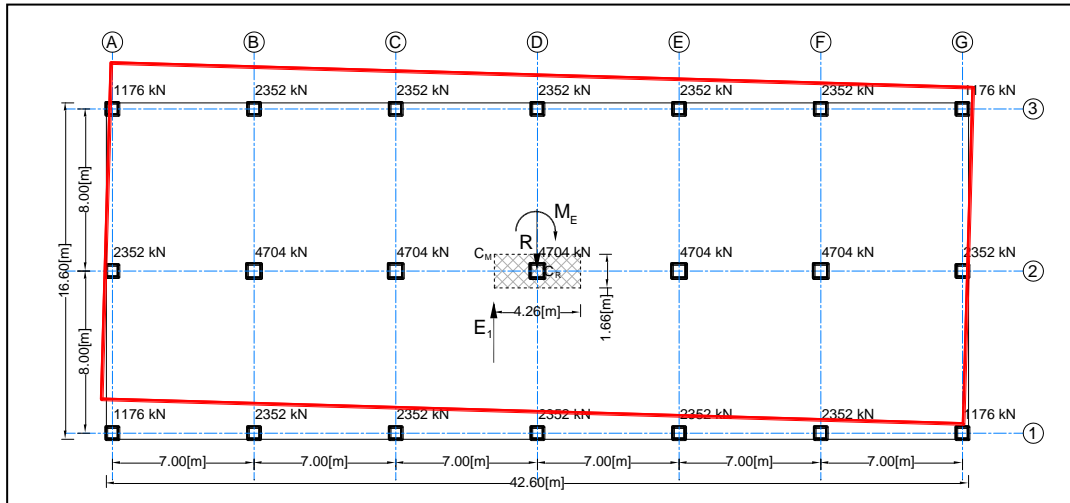


Figure 1. Building plan view

The elastomeric isolators are modelled with “rubber isolator” standard link element in Sap2000 computer program for laminated rubber bearings and multi-linear plastic links for no laminated isolator. For each shape factor the horizontal capacity of isolator is modelled with tri-linear curve in function of horizontal displacement. Both horizontal degrees of freedom are modelled with nonlinear behaviour. The finite element model for computational analyses is given in figure 2. The special points with number 50 to 57 are placed 2.13 meters to the left of building centre. To these points are attached the mass of each storeys level according to three degrees of freedom respectively; mass in “X” & “Y” direction and mass moment of inertia about “Z” axis [7].

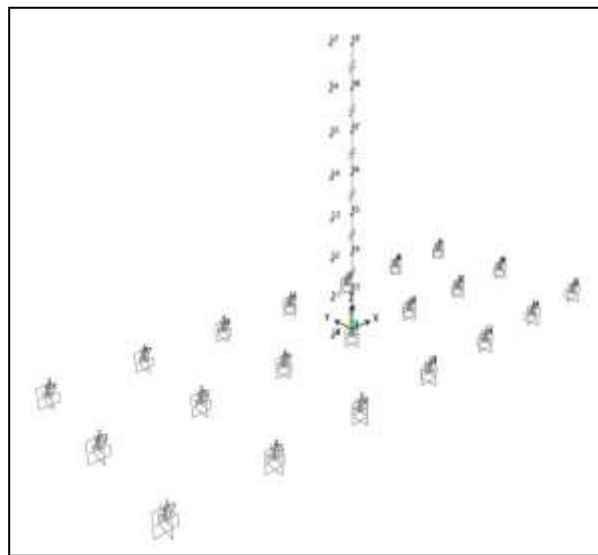


Figure 2. Finite elements model

The parameters for link elements presenting the building’s storeys are derived as below.

$$\text{Horizontal stiffness in “X” \& “Y” direction} \quad \rightarrow \quad K_H = \sum k_i \text{ with } k_i = 12EI_i/H^3 \quad (1)$$

$$\text{Torsional stiffness in “Z” direction} \quad \rightarrow \quad K_{RZ} = \sum (k_i x_i^2 + k_i y_i^2) \quad (2)$$

where

- E → module of elasticity for concrete material
- I_i → moment of inertia for column cross-section
- H → clear height of column for each storey
- x_i, y_i → coordinates of columns from rigidity centre of the storey

The stiffness of girders and slabs is assume infinite and so for each level storey are taken three degree of freedoms, two laterals and one torsional. Adequate constrains are put for each storey level and among the link elements modelling the isolator units.

The rubber compound for the production of elastomeric isolators used in our study is chosen the MVBR-0468 (X 0.4R) produced by Bridgestone with certification on December 2012. This material exercises a high dumping ratio and normally there is no need of additional dumper [6].

The effective dynamic shear modulus G_{eff} and the equivalent dumping ζ are function to shear strain. In our case the values for these parameters are given by the producer at shear strain $\gamma=1$ respectively $G_{eff}=0.392$ MPa and $\zeta=22\%$. Both parameters are sensitive to shear strains and their relationships to γ are given from the following expressions. For small values of γ the elastomeric isolator is stiff enough to avoid excessive displacements under low dynamic intensity of external loads such as wind [6].

$$G_{eff}(\gamma) = 0.054\gamma^4 - 0.416\gamma^3 + 1.192\gamma^2 - 1.583\gamma + 1.145 \quad (3)$$

$$\zeta_{eff}(\gamma) = -0.006\gamma^3 + 0.018\gamma^2 - 0.008\gamma + 0.216 \quad (4)$$

These values are true for no axial stress applied in isolator unit. In our case of study, the axial force can't be neglected and proper corrections must be done when calculating the lateral capacities of isolator unite.

Firstly we select the rubber bearing isolators depending on vertical static loads and the desire effective period of 2.5 s and target shear strain $\gamma=1.5$. The desired effective stiffness can be derived from the following expression [1]:

$$k_{eff} = 4 * \pi^2 * m / T_{eff}^2 \quad (5)$$

where "m" denotes the seismic mass supported by the isolator unit equal to N/g. "N" is the static compression axial load acting on isolator unit.

Two isolators with different vertical load capacity will be chosen, five isolator units with bearing axial load greater or equal to 4704 kN (group A) and sixteen isolator units with bearing capacity greater than 2352 kN (group B). Grouping the isolators make them more economical and easy to implement in site. The desired effective stiffness are respectively:

$$k_{eff}^{(A)} = 4 * 3.14^2 * 480 / 2.5^2 = 3029 \text{ kN/m}$$

$$k_{eff}^{(B)} = 4 * 3.14^2 * 240 / 2.5^2 = 1514 \text{ kN/m}$$

The effective stiffness can be calculated using the simplify formula considering only the shear deformation:

$$k_{eff} = G_{eff} * A / t_r \quad (6)$$

For $\gamma=1.5$ we have the effective shear modulus $G_{eff} = 322$ kPa and assuming the total height of the rubber $t_r=160$ mm, the cross-section area of the isolator result:

$$A^{(A)} = k_{eff}^{(A)} * t_r / G_{eff} = 3029 * 0.160 / 322 = 1.5051 \text{ m}^2$$

choosing the diameter $D = 1.3$ m with $A^{(A)} = 1.3249 \text{ m}^2$ and $k_{eff}^{(A)} = 2666.4 \text{ kN/m}$

$$A^{(B)} = k_{eff}^{(B)} * t_r / G_{eff} = 1514 * 0.160 / 322 = 0.7523 \text{ m}^2$$

choosing the diameter $D = 1.0$ m with $A^{(B)} = 0.7849 \text{ m}^2$ and $k_{eff}^{(B)} = 1579.6 \text{ kN/m}$

The effective stiffness of isolation system for horizontal directions is calculated as sum of single devices.

$$K_{eff} = 5 * k_{eff}^{(A)} + 16 * k_{eff}^{(B)} = 5 * 2666.4 + 14 * 1579.6 = 35446.4 \text{ kN/m.}$$

The effective period of isolation system is defined from the following expression:

$$T_{eff} = 2 * \pi * \sqrt{(M / K_{eff})} = 2 * 3.14 * \sqrt{(5754 / 35446.4)} = 2.53\text{s} \quad (4)$$

The above values correspond to unloaded isolators with axial force. The values, such as horizontal stiffness, are a function of critical vertical load, axial stress and shear strain. The critical load depends on first and second shape factors of isolator unit. The values of critical load and shape factors for the assumed rubber material are given from the following expressions [6]:

$$K_{eff,H} = K_{eff,H}^0 [1 - (\sigma_{sd} / \sigma_{cr}')^2] \quad (7)$$

$$\sigma_{cr}'(\gamma) = \sigma_{cr} * (1 - \gamma / S_2) \quad (8)$$

$$\sigma_{cr} = \alpha_c * \pi/4 * (G_{eq} * E_b)^{0.5} * S_2 \quad (9)$$

$$E_b = E_{cr} (1 + 2/3 * \kappa * S_1^2) / \{1 + E_{cr} (1 + 2/3 * \kappa * S_1^2) / E_{\infty}\} \quad (10)$$

- σ_{sd} → axial stress at isolator unit
 $\sigma_{cr}'(\gamma)$ → critical stress at γ strain
 σ_{cr} → critical stress at $\gamma = 0$
 $\alpha_c = 0.88 (1 - 0.07(5 - S_2))$ for $S_2 < 5$ or $\alpha_c = 0.88$ otherwise → correction factor
 $S_1 = D / (4 * t)$ → first shape factor
 $S_2 = D / t_r$ → second shape factor
 $E_{cr} = 3 * G_{eq}(\gamma=1)$ → critical modulus
 $E_o = 1300 \text{ MPa}$ → bulk modulus
 $\kappa = 0.223$ → isolator coefficient

For a given isolator unit the second shape factor is invariant. The first shape factor can varied in wide range and influence the critical stress by its quadratic value. To observe the influence of first shape factor let analyse the case of no lamination till a thickness of rubber layer equal to 8mm. The properties of isolator are given in tabular data in the table 1 & table 2.

TABLE I. ISOLATOR CHARACTERISTICS OF GROUP A

Isolator unit	Type - A						
	1	2	4	8	12	16	20
Nr. of layers							
Layer's thickness (mm)	160	80	40	20	13.3	10	8
First shape factor S_1	2.03	4.06	8.13	16.25	24.44	32.5	40
Critical stress at $\gamma = 0$	7901	11060	18990	35910	52770	68280	82960

TABLE II. ISOLATOR CHARACTERISTICS OF GROUP B

Isolator unit	Type - B							
	1	2	4	8	12	16	20	24
Nr. of layers								
Layer's thickness (mm)	160	80	40	20	13.3	10	8	6.7
First shape factor S_1	1.56	3.12	6.25	12.5	18.75	25	31.25	37.3
Critical stress at $\gamma = 0$	5710	7423	12030	22300	32810	42690	52300	60810

For each isolator unit we calculate the design stress caused by the vertical load. Increasing the shear strains from zero till to limit values, the shear capacity curves will be built. The capacity shear force will be calculated from the following formulas [2]:

$$V_{Rd,H} = K_{eff,H} * \gamma * t_r \quad (10)$$

For isolators of group A, the axial stresses are equal to

$$\sigma_{Sd,A} = 4704 / 1.3249 = 3550 \text{ kN/m}^2$$

For isolators of group B, the axial stresses for corner isolators and the remain ones are equal to

$$\sigma_{Sd,Bc} = 1176 / 0.7849 = 1498 \text{ kN/m}^2$$

$$\sigma_{Sd,B} = 2352 / 0.7849 = 2997 \text{ kN/m}^2$$

Keeping the axial stresses constant and reducing the critical stresses for variation of shear strain γ , the horizontal capacity curves are built for different values of first shape factor. The graphical presentation is given in figure 3 & figure 4. Also we make a nonlinear analysis without the reduction of horizontal stiffness. In this case the isolators are modelled with a built-in rubber isolator link element as describe in Sap2000 Manual. Neglecting the reduction of horizontal stiffness, as we will see, yield in lower base displacements and in higher base shear forces.

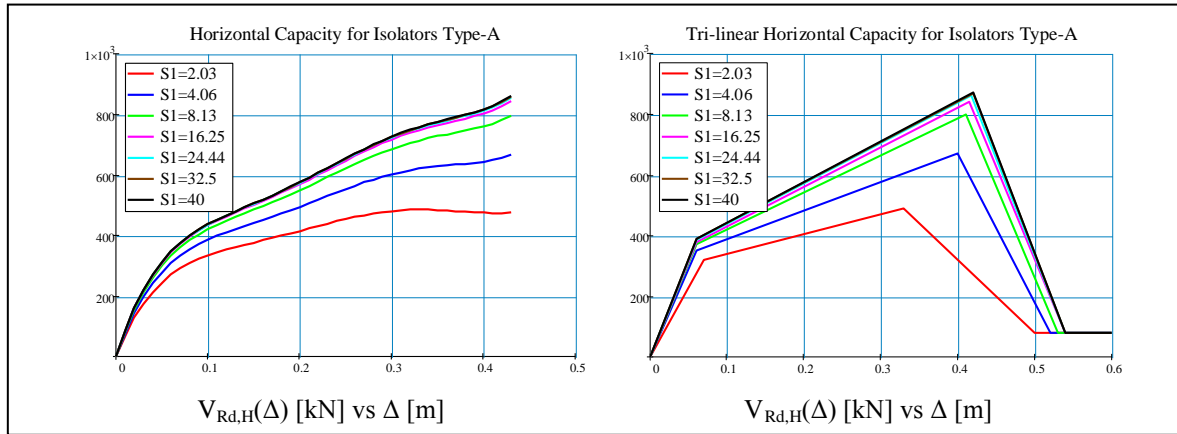


Figure 3. Capacity curves for horizontal direction of Type-A isolators

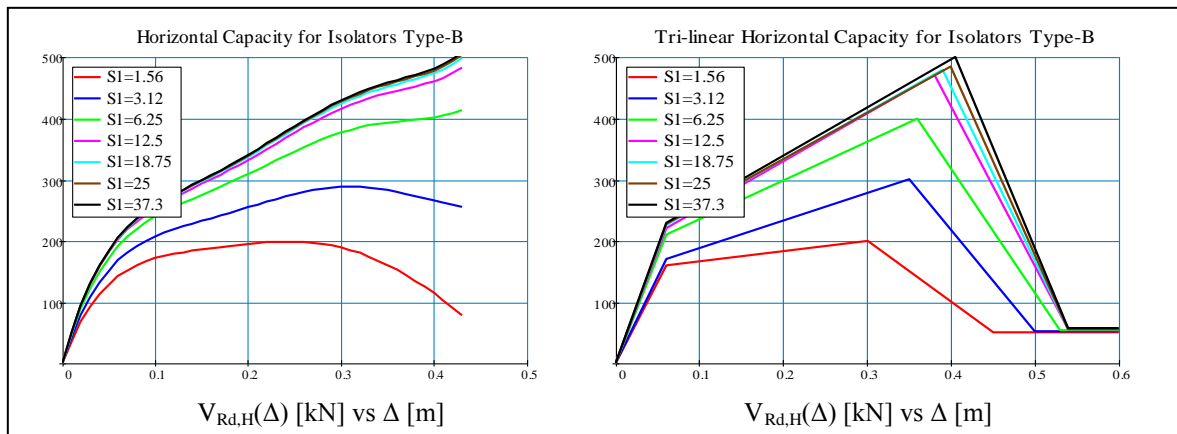


Figure 4. Capacity curves for horizontal direction of Type-B isolators

III. COMPUTATIONAL ANALYSES

The effect of vertical loads is included in calculation of the lateral capacity curves for each isolator unit. To simulate the lateral loads, the Santa Monica earthquake accelerogram is taken as shown in figure 5.

The response of isolated building is done through nonlinear time histories analyses. Two kinds of nonlinear THS are performed, fast nonlinear analyzing using the modal values and direct integration. The modal analyses are done using the Ritz vectors including all isolators unit and two lateral acceleration in “X” and “Y” direction with target accuracy 99%. Using this approach the FNA and NLDIH gives almost the same results [7].

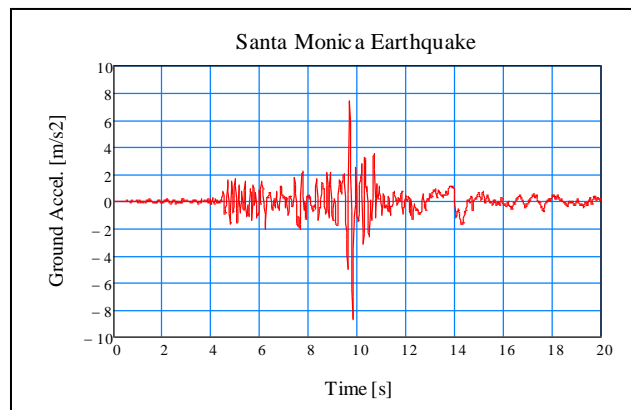


Figure 5. Santa Monica earthquake accelerogram

From the analyses the translative displacement are taken excluding the rotational degree of freedom to vertical axis. These displacement are given in table 3, and as can be seen the first shape factor influence is not linear. For shape factor over the value 8 there is not a significant change in displacement. This result must not be taken as adequate reminding that the axial load on isolator is kept constant.

TABLE III. STOREYS DISPLACEMENT WITHOUT TORSION

Storey Level	1 layer	2 layers	4 layers	20 layers	No axial load
Ground	0	0	0	0	0
Base Isolation	29.4	24.3	19.3	19.1	13.8
Storey – 1	29.6	24.5	19.6	19.4	14.1
Storey – 2	29.7	24.8	19.8	19.6	14.2
Storey – 3	29.9	24.9	20.1	19.8	14.4
Storey – 4	30.0	25.1	20.3	20.0	14.5
Storey – 5	30.1	25.3	20.4	20.2	14.6
Storey – 6	30.2	25.4	20.5	20.3	14.6
Storey – 7	30.3	25.5	20.6	20.4	14.7

In figure 6 are presented graphically the translative displacements of the base isolation and for each storey. Also there is shown the scheme of plan displacement of base isolation when the torsional degree of freedom is included.

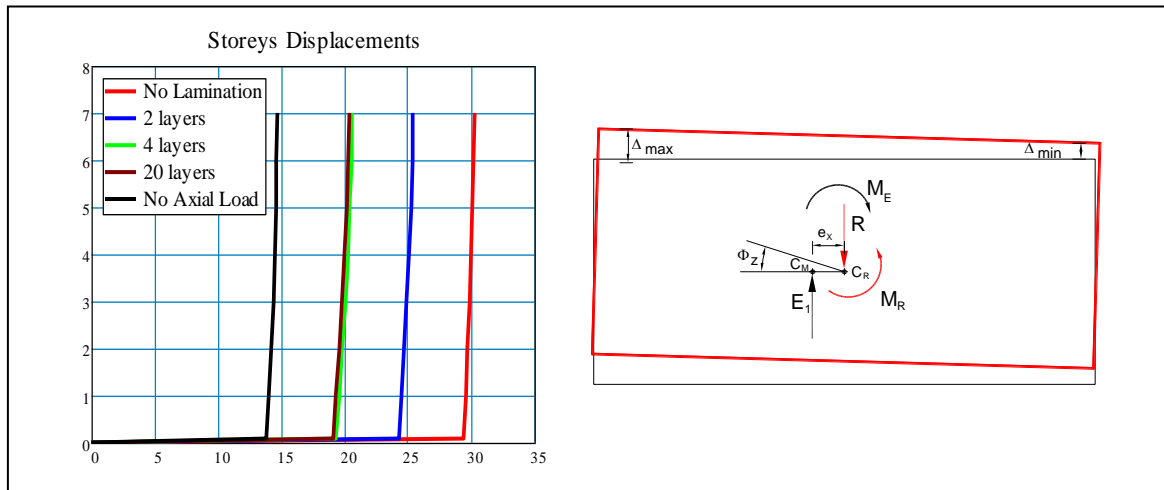


Figure 6. Santa Monica earthquake accelerogram

In table 4 are given the maximal and minimal displacements of base isolation system at maximum lateral displacement for different values of first shape factor. At these deformed states the base shear forces and torsional moments to vertical axis are given. The eccentricity is calculated as ratio of torsional moment to base shear force. As it can be seen, the eccentricity is not a fix value and has a wide variation. The magnitude of these variations depends on initial eccentricity, rotational mass moment of inertia, first shape factor, magnitude of axial load on isolator unit, external lateral forces etc.

TABLE IV. MAXIMAL AND MINIMAL DISPLACEMENTS. VARIATION OF ECCENTRICITY

Nr. Layers	$\Delta-max$	$\Delta-min$	$\Phi-z$	V_y	M_z	$e-x$
1	36.2	8.4	0.38	5064	46830	9.25
2	32.6	12.9	0.27	6389	36930	5.78
4	22.3	16.3	0.08	7138	5815	0.81
20	20.4	16.6	0.05	8079	38793	4.80
No axial load	18.6	8.1	0.14	7796	65570	8.41

In figure 7 are given the time histories of displacements for the left-corner isolator which expresses higher values of strains. The base isolation system consists of no laminated isolators has larger displacements and there is a lack of restoring force. For the base isolation system with no axial forces in isolator units, the maximal displacement doesn't differ too much with the case of full laminated, but the displacements are dumped faster.

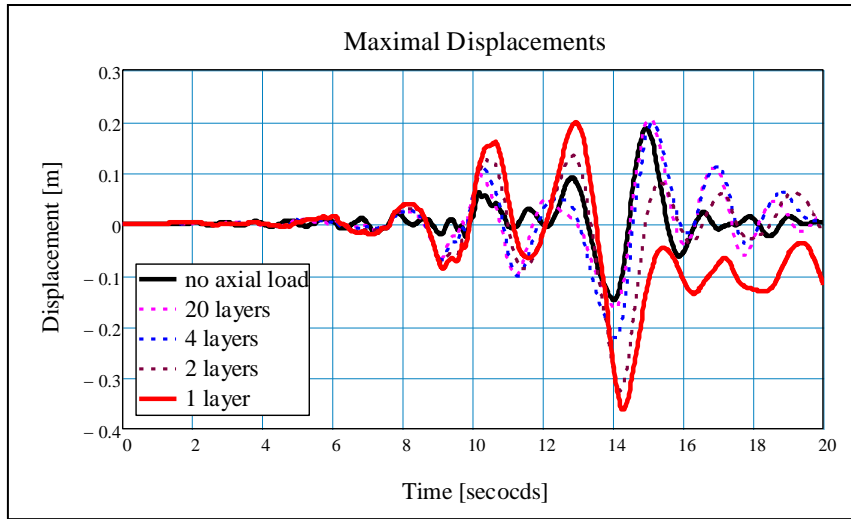


Figure 7. Santa Monica earthquake accelerogram

In figure 8 are given the hysteresis curves for left-corner isolators. It can be clearly seen that the effective stiffness depend directly to the amount of laminated layers and magnitude of axial load. The lack of restoring force can be observed for the no laminated isolator.

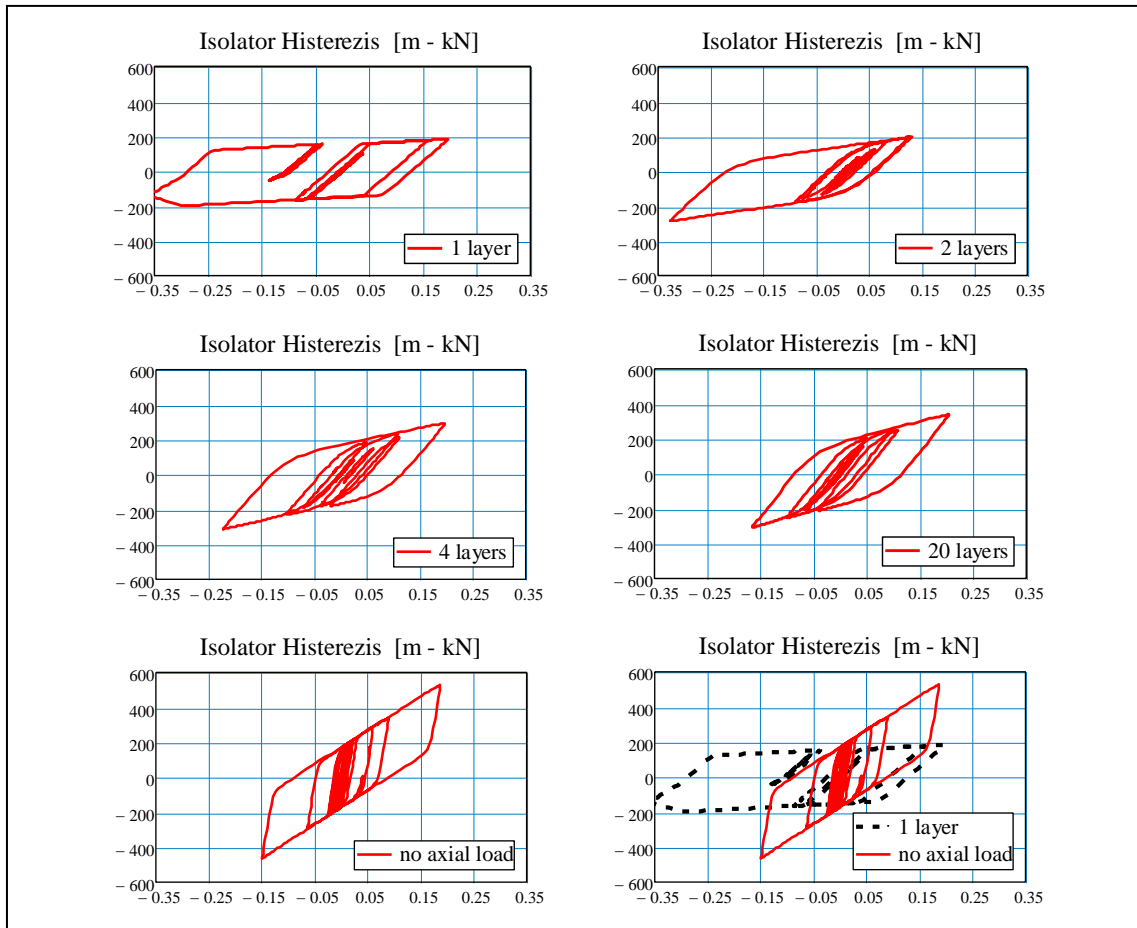


Figure 8. Santa Monica earthquake accelerogram

IV. CONCLUSIONS

Rubber bearing base isolation system behaviour depends directly by the first shape factor. In accordance to the magnitude of axial load acting upon the isolator unit, certain value of first shape factor must not be allowed. In this study the axial load are taken as invariant and the minimum acceptable value for the shape factor is around 10. In reality the axial load vary during the time history due to overturning moment, especially in peripheral isolator units and so a higher value for first shape factor is required. Other requirements for vertical stiffness might yield in higher value of shape factor. For low values of first shape factor the displacements increase causing large shear strain. In such strains the restoring force is not stable and can yield in unit failure. Even in symmetrical building, the existence of accidental eccentricity of mass, yield in considerable torsional moment. This torsional moment induces different displacement in isolator units and hence the isolator units manifest different effective stiffness. This phenomenon increase the initial eccentricity and gives a higher torsion, so isolation system exercise additional displacement till the internal equilibrium is achieved. The eccentricity vary during the time history and depend from many parameters such as building geometry, initial eccentricity, magnitude of external lateral force, but is more sensitive for low values of lamination. Higher values of first shape factor give a stiffer base isolation system. Although the additional energy transited in superstructure is not considerable, only few additional drifts can result in first floor of superstructure. In this study the fluctuation of rubber properties due to environmental condition such as temperature oscillations or time depended properties from aging are not consider. The lamination gives a good confinement to rubber and reduces the degradation of the material. This is another issue in evaluation of first shape factor in life durability of base isolation systems.

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