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A Bayesian Approach to Stochastic Claims Reserve Estimation

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Abstract: **The Chain-ladder techniques are conventional and distribution free methods used to estimate stochastic reserving in non-life insurance. But the results produced by the Chain-ladder methods do not adequately consider high fluctuations arise in the data due to large claims. The alternative way to accommodate such extreme and influenced values is possible through probability distributions especially through Bayesian framework. This paper presents certain results on estimation of parameters of probability models used in stochastic claims reserving by using simulation techniques.**

Keywords: Chain Ladder Method; Stochastic Claims Reserving; IBNR; MCMC

I. Introduction

The chain-ladder techniques are deterministic method for predicting claim amounts by the Actuarial professional. Further it is designed to predict future incremental claim amounts based on the available claims data at hand. One of the serious shortcomings of this method is that it does have provision for conducting diagnostic checks and inability to estimate the confidence intervals. Hence stochastic claim reserving techniques were developed to have advantages over deterministic techniques on these counts. It also uses the information gained from statistical reserving methods. Later Thomas Mack measure the variability of chain ladder reserve estimates and construct a confidence interval for the estimated reserves.

While discussing stochastic reserving, it is important to differentiate between a model and the approach which is used to implement this model. In this context, the model is a statistical model describing the underlying insurance claim process and the approach is a method/technique used to deploy such a model in a particular reserving situation (e.g. assessing uncertainties inherent in the past data). Unfortunately, there is no single model that suits all reserving problems. A robust approach should be flexible enough to accommodate any suitable model and perform well in all reserving situations.

Further stochastic reserving is an attempt to quantify this uncertainty and estimate the distribution underlying the reserves for a particular insurance portfolio. The scope of stochastic reserving is broader than for traditional reserving methods, which are generally concerned with the estimation of a central estimate. In order to address the problem of uncertainty and prior information about the claims process, the Bayesian stochastic reserving will rescue the insurance practitioner from these hurdles. For Bayesian stochastic reserving more in details the reader may refer the papers of de Alba(2002) , England and Verrall (2002), Ntzoufras and Dellaportas (2002), Scollnik(2004), Verrall (2004) and Verrall and England (2005). In this paper, we present application Bayesian Modeling for estimation of stochastic claim reserve estimates using simulation techniques.

II. Bayesian Modeling Procedure

Step 1 : Specify of Sampling Distribution
(i.e.,) Specify a probability distribution for the data with some unknown parameters.

Step 2 : Specify Prior Distribution
(i.e.,) Specify prior probability distributions for the parameters of the sampling distribution.

Step 3 : Obtain the Likelihood Function
(i.e.,) Derive the likelihood function of the parameters for the given the data.

Step 4: Obtain Posterior Distribution
(i.e.,)Combine the likelihood function with the prior distributions to derive the posterior joint distribution of the parameters for the data under consideration.

Step 5 : Estimation of parameters of Posterior Distribution
(i.e.,)Obtain estimates of unknown parameters of the posterior distribution

Step 6 : Obtain the Predictive Distribution
(i.e.,) Obtain forecasts using the predictive distribution derived by combining the posterior distribution with the prior distributions

III. Chain-Ladder Method

Let us assume that the data consist of a triangle of incremental claims. This is the simplest shape of data that can be obtained, and it is often the case that data from early origin years are considered fully run-off or that other parts of the triangle are missing. We assume that we have the following set of incremental claims data:

$$\left\{C_{ij} : i = 1, 2, ..., n;\; j = 1, 2, ..., n - i + 1\right\}$$

The suffix *i* refers to the row, and could indicate accident year or underwriting year. The suffix *j* refers to the column, and indicates the delay, here assumed also to be measured in years. The skeleton of the table is presented as follows

Origin Period	*Development Period* 1	2	3	…	n
1	c_{11}	c_{12}	c_{13}	…	c_{1n}
2	c_{21}	c_{22}	c_{23}	…	c_{2n}
3	c_{31}	c_{32}	c_{33}	…	c_{3n}
…	…	…	…	…	…
n	c_{n1}	c_{n2}	c_{n3}	…	c_{nn}

The cumulative claims are defined by:

$$D_{ij} = \sum_{k=1}^{j} C_{ik}$$

And the development factors of the chain-ladder technique are denoted by $\left\{\lambda_j ;\, j = 2, ..., n\right\}$. The Chain-ladder technique estimates the development factors as:

$$\hat{\lambda} = \frac{\sum_{i=1}^{n-j+1} D_{ij}}{\sum_{i=1}^{n-j+1} D_{i,\, j-1}}.$$

These are then applied to the latest cumulative claims in each row to produce forecasts of future values of cumulative claims. Thus, the chain-ladder technique, in its simplest form, consists of a way of obtaining forecasts of ultimate claims only. Here `ultimate' is interpreted as the latest delay year so far observed, and does not include any tail factors. From a statistical viewpoint, given a point estimate, the natural next step is to develop estimates of the likely variability in the outcome so that assessments can be made, for example, of whether extra reserves should be held for prudence, over and above the predicted values. In this respect, the measure of variability commonly used is the prediction error, defined as the standard deviation of the distribution of possible reserve outcomes. It is also desirable to take account of other factors, such as the possibility of unforeseen events occurring which might increase the uncertainty, but which are difficult to model.

The reserving problem can be approached as follows:

Steps 1 & 2: Assume that each C_{ij} follows a probability distribution $f\left(c_{ij}/\theta\right)$, where θ denotes a singular or vector of parameters describing a particular claim process generating C_{ij}, and all parameter(s) are distributed according to a prior distribution function $\pi(\theta)$.

Steps 3: Calculate the likelihood function $L(\theta/c)$ for the parameters given the observed data:

$$L(\theta/\underline{c}) = \prod_{i+j\leq n+1} f\left(c_{ij}/\theta\right)$$

Steps 4: Given the data distribution and the prior distribution, the posterior distribution $f(\theta/c)$ is proportional to the product of the likelihood function and the prior distribution.

Steps 5: Parameters θ are obtained from $f(\theta/c)$ and used in the next step.

$$f(\theta/\underline{c}) \propto L(\theta/\underline{c})\pi(\theta)$$

Step 6: As noted by de Alba (2002), if we were interested in inference about the parameters θ we could end our modelling process at step 5 and look at the properties of $f(\boldsymbol{\theta}/\mathbf{c})$.

However if our aim is prediction, as in the case of stochastic reserving, then the known data C_{ij} (for $i+j \leq n+1$) is used to predict unobserved values in the lower-right triangle C_{ij} (for $i+j > n+1$) by means of the predictive distribution.

$$f\left(c_{ij}/\theta\right) = \int f\left(c_{ij}/\theta\right) f(\theta/\underline{c})\,d\theta$$

for $i, j = 1,2,\ldots,n$ and $i+j>n+1$

IV. Application of Bayesian Stochastic Reserving in Chain-ladder model

Consider the following data for the estimation of Bayesian stochastic reserving by using the term ultimate loss ratio. The following table shows that outstanding claims column donates aggregate of the fully settled and partial settled claims of current and past years. The column Earned premium denotes total of all insurance policy premium which at active at present. The last column ultimate loss ratio denotes the percentage of total liquidity used for fully settled and partially settled claims of the current year.

Historical loss development study (1991) Automatic Facultative

General Liability data

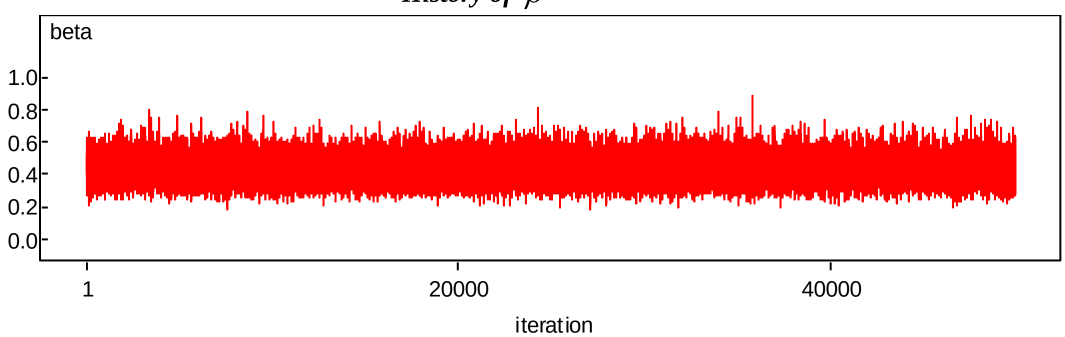
Year	1	2	3	4	5	6	7	8	9	10	Outstan-ding claims	Earned Premium	Ultimate Loss Ratio
1	5,012	3,257	2,638	898	1734	2642	1828	599	54	172	0	28975	65%
2	106	4,179	1111	5270	3116	1817	-103	673	535		154	20478	82%
3	3,410	5,582	4881	2268	2594	3479	649	603			617	28984	83%
4	5,655	5,900	4211	5500	2159	2658	984				1636	38432	75%
5	1,092	8,473	6271	6333	3786	225					2747	47290	61%
6	1,513	4,932	5257	1233	2917						3649	24308	80%
7	557	3,463	6926	1368							5435	23228	76%
8	1,351	5,596	6165								10907	30721	78%
9	3,133	2,262									10650	29611	54%
10	2,063										16339	29407	63%
Chain ladder factors		2.999	1.624	1.271	1.172	1.113	1.042	1.033	1.017	1.009			

Here, our interest is to find the estimate of future stochastic reserve for a company to maintain to avoid solvency by Bayesian methodology with help of winBUGS software.

Here, Let us assume that the sampling distribution of the ultimate loss ratio (X) follows Pareto distribution with parameters α and β. The prior distribution of the parameter (β) is gamma distribution which is conjugate prior but it is also more appropriate for this stochastic reserving problem. Let x ~ Pareto(α, β) and β ~ gamma(a, b) then the posterior distribution of β is obtained by using simulation technique with the help of winBUGS software.

winBUGS output : For the values of n =10, α = 2, a = 30 and b = 75

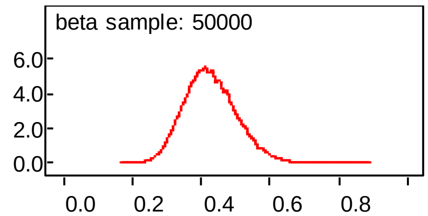
History of β



Statistics

Node	mean	sd	MC error	2.5%	median	97.5%	start	sample
beta	0.4267	0.07559	3.544E-4	0.2928	0.4218	0.5873	1	50000

Density of β



From the above output, it is observed that the suggested future claims reserve of a company should be 85.34% of the earned premium. And this value is likely to change for different set of values of parameters involved in the prior distribution.

V. Summary and finding

The choice of selecting an appropriate statistical model given a particular reserving situation is outside the scope of this paper. This paper presented a high-level outline of the key concepts and mechanisms of Bayesian stochastic reserving. Further, it illustrates the Bayesian stochastic reserving when the real time data is presented which is partly available in reality and the rest of the values can be obtained by the method of chain ladder is discussed. There are plenty of research work is yet unearth in this area. Further, the Bayesian stochastic reserving is tip of the ice berg and the world of uncertainty can restricted by assumptions and predicted by statistical models enormously and endlessly.

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