(IJIRSE) International Journal of Innovative Research in Science & Engineering
ISSN (Online) 2347-3207

Solving Multi-Objective Structural Design Problem using Fuzzy Optimization Method: A Comparative Study

Samir Dey
Department of Mathematics, Asansol Engineering College
Vivekananda Sarani, Asansol-713305, West Bengal, India.
samir_besus@rediffmail.com

Tapan Kumar Roy
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur.
P.O.-Botanic Garden, Howrah-711103, West Bengal, India.
roy_t_k@yahoo.co.in

Abstract— **Real world engineering design problems are usually characterized by the presence of many conflicting objectives. The paper aims to give computational algorithm to solve a multi-objective structural problem using fuzzy optimization method. The development of algorithm is based on principle of optimal decision set obtained by intersection of various fuzzy decision sets which are obtained corresponding to each objective function. We made a comparative study of linear and non-linear membership function to see its impact on optimization and to get to the depth of such optimization process. A classical truss optimization example is presented here to demonstrate the efficiency of our propose optimization approach. The test problem includes a three-bar planar truss subjected to a single load condition. This approximation approach is used to solve this multi-objective structural optimization model. The model is illustrated with the help of numerical examples.**

Keywords- Multi-objective Structural Problem, Fuzzy Programming Algorithm, Hyperbolic Membership Function, Linear Membership Function, Optimization Problem.

I. INTRODUCTION

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker. Decision making is based solely on a single criterion appears insufficient as soon as the decision-making process deals with the complex environment. So, one must acknowledge the presence of several criteria that lead to the development of multi-criteria decision making.

Structural Optimization is a kind of decision making, in which decision have to be made to optimize one or more objectives under prescribed set of circumstances. These structural problem may be single or multi-objective and are to be optimized (maximized or minimized) under a specified set of constraints. The main objective of structural engineering is to design structures which can withstand external loads safely and at a minimum cost or weight.

Different solutions may produce trade-offs (conflicting scenarios) among different objectives. A solution that is extreme (in a better sense) with respect to an objective requires a compromise in other objectives. In multi-objective optimization, there may not exist a solution that is best with respect to all objectives. Instead, there are equally good solutions which are known as Pareto optimal solutions. A Pareto optimal set of solution is such that when we go from any one point to another in the set, at least one objective function improves and at least one of the others worse. Neither of the solutions dominates over each other. All the sets of decision variables on the Pareto front are equally good and are expected to provide flexibility for the decision maker. Normally, the decision about “what the best answer is” corresponds to the so-called decision maker. Finally the book by Eschenauer et al.[3] is a very valuable guide to some of the most relevant work in multi-objective design optimization in the last few years. Good surveys on structural optimization may be in literatures [1,4-6,8-10]

In real life, the data cannot be recorded or collected precisely due to human errors or some unexpected situations. So one may consider ambiguous situations like vague parameters, non-exact objective and constraint functions in the problem and it may be classified as a non-stochastic imprecise model. Here fuzzy set theory may provide a method to describe or formulate this imprecise model. Zadeh [16] first gave the concept of fuzzy set theory for handling uncertainty that is due to imprecision rather than to randomness. Later on Bellman and Zadeh [7] used the fuzzy set theory for decision making problem. Zimmermann [17] proposed a fuzzy multi-criteria decision making set, defined as the intersection of all fuzzy goals and their constraints.

In practical, the problem of structural design may be formed as a typical non-linear programming problem with non-linear objective functions and constraints functions in fuzzy environment. Some researchers applied the fuzzy set theory for Structural model. For example Wang et al. [13] first applied α -cut method to structural designs where the non-linear problems were solved with various design levels α , and then a sequence of solutions are obtained by setting different level-cut value of α . Rao et al.[8-9] showed significant work in multi-objective structural optimization with uncertain parameters. Rao [10] applied the same α -cut method to design a four–bar mechanism for function generating problem .Structural optimization with fuzzy parameters was developed by Yeh et.al [15]. In 1989, Xu [14] used two-phase method for fuzzy optimization of structures. In 2004, Shih et.al [12] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources .Shih et.al [11] developed an alternative α -level-cuts methods for optimum structural design with fuzzy resources in 2003. Dey et.al [2] optimize structural model in fuzzy environment.

The motivation of the present study is to give computational algorithm for solving multi-objective nonlinear programming problem by fuzzy optimization approach. We also aim at studying the impact of various type of membership functions in such optimization process and thus made comparative study of linear and nonlinear membership.

The remainder of this paper is organized in the following way. In section 2, we discuss about structural optimization model. In section 3, we discuss about mathematics Prerequisites. Section 4 contains basic principle of fuzzy optimization needed for developing algorithm. Section 5 contains two computational algorithms and the algorithm has been implemented on an illustration in section 6. The result obtained has been placed in section 7 followed by references.

II. Multi-objective Structural Model

In the design of optimal structure i.e. lightest weight of the structure and minimum deflection of loaded joint that satisfies all stress constraints in members of the structure. To bar truss structure system the basic parameters (including the elastic modulus, material density, the maximum allowable stress, etc.) are known and the optimization’s target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight, the minimum nodes displacement, in a given load conditions.

The multi-objective Structural model can be expressed as:

$$\begin{aligned} &minimize\ WT(A) \\ &minimize\ \delta(A) \\ subject\ to\quad &\sigma(A) \le [\sigma] \\ &A_{\min} \le A \le A_{\max} \end{aligned} \tag{1}$$

where $A = \left[A_1, A_2, ..., A_n\right]^T$ are design variables for the cross section, n is the group number of design variables for the cross section bar, $WT\left(A\right) = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure, $\delta(A)$ is the deflection of loaded joint, L_i , A_i and ρ_i were the bar length, cross section area, and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $\left[\sigma\right]$ is maximum allowable stress of the group bars under various conditions, $A_{\min}$ and $A_{\max}$ are the minimum and maximum cross section area respectively.

III. Prerequisite Mathematics

A. *Fuzzy Set*

Let X is a set (space), with a generic element of X denoted by x , that is $X(x)$. Then a Fuzzy set (FS) is defined as $A = \left\{\left(x, \mu_A(x)\right) : x \in X\right\}$ where $\mu_A : X \to [0,1]$ is the membership function of FS A . $\mu_A(x)$ is the degree of membership of the element x to the set A .

B. *α -Level Set or α -cut of a Fuzzy Set*

The α -level set of the fuzzy set A of X is a crisp set A_α that contains all the elements of X that have membership values greater than or equal to α i.e. $A = \left\{x :\ \mu_A(x) \ge \alpha,\ x \in X, \alpha \in [0,1]\right\}$.

IV. Mathematical Analysis

A. General Fuzzy Non-linear Programming Technique to Solve Multi-objective Non-linear Programming Problem(MONLP)

A Multi-Objective Non-Linear Programming (MONPL) or Vector Minimization problem (VMP) may be taken in the following form:

$$Min\ f(x) = [f_1(x), f_2(x), \dots\dots\dots f_k(x)]^T \tag{2}$$

Subject to $x \in X = \left\{ x \in R^n : g_j(x) \le or = or \ge b_j \ \ for \ \ j = 1,2,3,\dots.,m \right\}$ and $l_i \le x_i \le u_i \quad (i = 1,2,3,\dots.,n)$.

Zimmermann (1978) showed that fuzzy programming technique can be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

Step 1: Solve the MONLP (3) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

Step 2: From the result of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \dots. \\ x^k \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \dots. & f_k(x) \end{array} \\ \begin{pmatrix} f_1^*(x^1) & f_2^*(x^1) & \dots. & f_k^*(x^1) \\ f_1^*(x^2) & f_2^*(x^2) & \dots. & f_k^*(x^2) \\ \dots. & \dots. & \dots. & \dots. \\ f_1^*(x^k) & f_2^*(x^k) & & f_k^*(x^k) \end{pmatrix} \end{array}$$

Here x^1 x^2, x^3,….., x^k are the ideal solutions of the objectives $f_1(x)$, $f_2(x)$,….., $f_k(x)$ respectively. The maximum value of each column U_r gives upper bound or upper tolerance or highest acceptable level of achievement for the r^{th} objective function $f_r(x)$, where $U_r = \max\left\{ f_r(x^1), f_r(x^2), \dots\dots, f_r(x^k) \right\}$ and the minimum value of each column L_r gives lower bound or lower tolerance limit or aspired level of achievement for the r^{th} objective function $f_r(x)$ where $L_r = \min\left\{ f_r(x^1), f_r(x^2), \dots\dots, f_r(x^k) \right\}$ for $r = 1,2,\dots.,k$.

Step 3: Using aspiration level of each objective of the MONLP (3) may be written as follows:

Find x so as to satisfy

$$f_r(x) \le L_r \ \text{ with tolerance } \ P_r\left(= U_r - L_r\right) \quad \text{for} \quad r = 1,2,3,\dots\dots..,k$$

$$x \in X. \quad l_i \le x_i \le u_i \quad (i = 1,2,3,\dots.,n)$$

Step 4. Define a membership function $\mu_r(f_r(x))$ for the r^{th} objective function.

Step 5. Convert the fuzzy mode of the problem, obtained in step, into the following crisp model.

$$\text{Maximize } \lambda \tag{3}$$

$$\text{subject to } \ \mu_r(f_r(x)) \ge \lambda, \text{ for } r = 1,2,3,\dots\dots..,k$$

$$\lambda \in [0,1], x \in X. \quad l_i \le x_i \le u_i \quad (i = 1,2,3,\dots.,n)$$

Step 6. Solve the crisp model by an appropriate mathematical programming algorithm.

Step 7. The solution obtained in step 6 will be the optimal compromise solution of the MONLP.

B. Membership Functions

One of the major assumptions in solving fuzzy mathematical programming problems in the literature involves the use of linear membership functions for all fuzzy sets involved in a decision making process. A linear approximation is most commonly used because of its simplicity and is defined by fixing two points, the upper and lower levels of acceptability.

Let U_r and L_r be the highest acceptable level of achievement and the aspired level of achievement for the r^{th} objective function, respectively. In the following two subsections we study different membership functions.

*B.1. Linear Membership Function***:** A linear membership function can be defined as follows

$$\mu_r^L(f_r(x)) = \begin{cases} 0 & if \;\; f_r(x) \ge U_r \\ \dfrac{U_r - f_r(x)}{U_r - L_r} & if \;\; L_r \le f_r(x) \le U_r \\ 1 & if \;\; f_r(x) \le L_r \end{cases}, \tag{4}$$

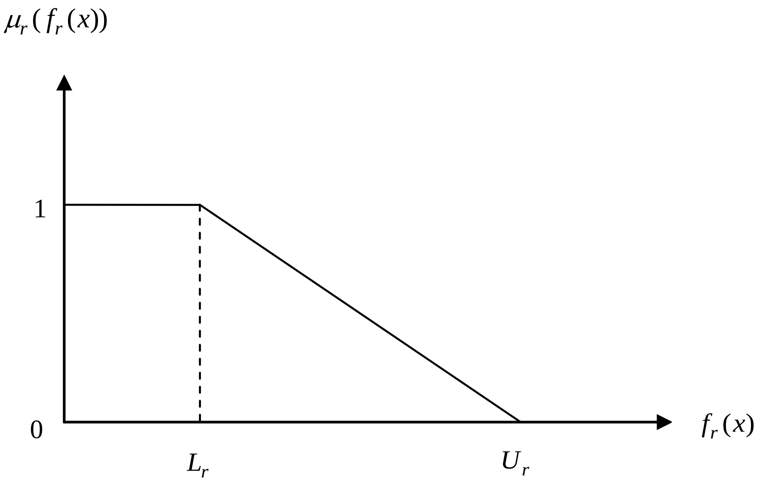

Figure 1: Linear Membership function for $f_r(x)$

*B.2. Hyperbolic Membership Function***:** A hyperbolic membership function is defined by

$$\mu_r^H(f_r(x)) = \begin{cases} 1 & if \;\; f_r(x) \le L_r \\ \dfrac{1}{2} - \dfrac{1}{2}\tanh\left[\left(\dfrac{U_r + L_r}{2} - f_r(x)\right)\alpha_r\right] & if \; L_r \le f_r(x) \le U_r \\ 0 & if \;\; f_r(x) \ge U_r \end{cases} \tag{5}$$

where $\alpha_r = \dfrac{6}{U_r - L_r}$.

This membership function has the following formal properties:

i) $\mu_r^H(f_r(x))$ is strictly monotonously decreasing function with respect to $f_r(x)$.

ii) $\mu_r^H(f_r(x)) = \dfrac{1}{2} \Leftrightarrow f_r(x) = \dfrac{1}{2}(U_r + L_r)$

iii) $\mu_r^H(f_r(x))$ is strictly convex for $f_r(x) \ge \dfrac{1}{2}(U_r + L_r)$ and strictly concave for $f_r(x) \le \dfrac{1}{2}(U_r + L_r)$.

iv) $\mu_r^H(f_r(x))$ satisfies $0 < \mu_r^H(f_r(x)) < 1$ for $L_r \le f_r(x) \le U_r$ and approaches asymptotically $\mu_r^H(f_r(x)) = 0$ and $\mu_r^H(f_r(x)) = 1$ as $f_r(x) \to \infty$ and $-\infty$, respectively.

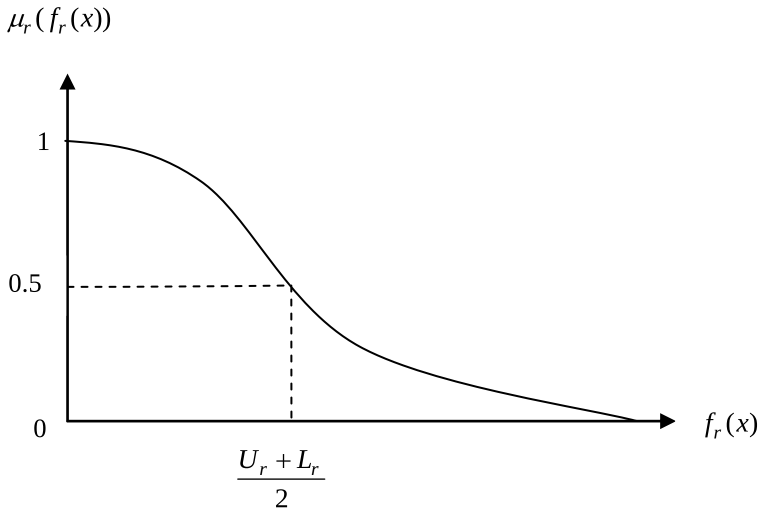

Figure 2: Hyperbolic Membership function for $f_r(x)$

V. Computation Algorithm for Multi-Objective Structural Problem

A. Algorithm I (Linear Membership Function)

Step 1. Taking the first objective function from set of objectives of the problem (1) and solve it as a single objective subject to the given constraints. Find the value of objective functions and decision variables.

Step 2.From values of these decision variables compute values of remaining objectives.

Step 3. Repeat the Step 1 and Step 2 for remaining objective functions.

Step 4. After that according to step 3 pay-off matrix formulated as follows:

$$\begin{array}{c c} & \begin{array}{cc} WT(A) & \delta(A) \end{array} \\ \begin{array}{c} A^1 \\ \\ A^2 \end{array} & \begin{pmatrix} WT^*(A^1) & \delta(A^1) \\ & \\ WT(A^2) & \delta^*(A^2) \end{pmatrix} \end{array}$$

Step 5. The bounds are $U_1 = \max\left\{WT\left(A^{1*}\right), WT\left(A^{2*}\right)\right\}$, $L_1 = \min\left\{WT\left(A^{1*}\right), WT\left(A^{2*}\right)\right\}$ for weight function $WT(A)$ (where $L_1 \le WT(A) \le U_1$) and the bounds of objective are $U_2 = \max\left\{\delta\left(A^{1*}\right), \delta\left(A^{2*}\right)\right\}$, $L_2 = \min\left\{\delta\left(A^{1*}\right), \delta\left(A^{2*}\right)\right\}$ for deflection function $\delta(A)$ (where $L_2 \le \delta(A) \le U_2$) are identified.

Step 6. Use following linear membership functions $\mu_{WT}\left(WT\left(A\right)\right)$ and $\mu_{\delta}\left(\delta\left(A\right)\right)$ for the objective functions $WT\left(A\right)$ and $\delta\left(A\right)$ respectively are defined as follows:

$$\mu_{WT}\left(WT\left(A\right)\right) = \begin{cases} 1 & if \;\; WT\left(A\right) \le L_1 \\ \left(\dfrac{U_1 - WT\left(A\right)}{U_1 - L_1}\right) & if \;\; L_1 \le WT\left(A\right) \le U_1 \\ 0 & if \;\; WT\left(A\right) \ge U_1 \end{cases}$$

$$\mu_{\delta}\left(\delta(A)\right)=\begin{cases}1 & if\ \ \delta(A)\le L_2\\ \left(\dfrac{U_2-\delta(A)}{U_2-L_2^{'}}\right) & if\ \ L_2\le\delta(A)\le U_2\\ 0 & if\ \ \delta(A)\ge U_2\end{cases}$$

Step 7. Now the fuzzy optimization method for MOSP problem (1) with linear membership function gives equivalent nonlinear programming problem as

$$\begin{aligned}&Maximize\ \ \lambda\\ &subject\ to\ WT(A)+\lambda(U_1-L_1)\le U_1\\ &\qquad\delta(A)+\lambda(U_2-L_2)\le U_2\\ &\qquad\sigma(A)\le[\sigma]\\ &\qquad A_{\min}\le A\le A_{\max},\lambda\in[0,1].\end{aligned}\tag{6}$$

Step 8. The above non-linear programming problem (6) can be easily solve an appropriate mathematical programming algorithm.

B. Algorithm II (Hyperbolic Membership Function)

Repeat steps 1 to step 5 and construct pay off matrix.

Step 6. Using hyperbolic membership function for each objective functions

$$\mu_{WT}\left(WT(A)\right)=\begin{cases}1 & if\ \ WT(A)\le L_1\\ \dfrac{1}{2}-\dfrac{1}{2}\tanh\left[\left(\dfrac{U_1+L_1}{2}-WT(A)\right)\alpha_1\right] & if\ L_1\le WT(A)\le U_1\\ 0 & if\ \ WT(A)\ge U_1\end{cases}$$

$$\mu_{\delta}\left(\delta(A)\right)=\begin{cases}1 & if\ \ \delta(A)\le L_2\\ \dfrac{1}{2}-\dfrac{1}{2}\tanh\left[\left(\dfrac{U_2+L_2}{2}-\delta(A)\right)\alpha_2\right] & if\ L_2\le\delta(A)\le U_2\\ 0 & if\ \ \delta(A)\ge U_2\end{cases}$$

where $\alpha_r=\dfrac{6}{U_r-L_r}$, $r=1,2$.

Step 7. Now the fuzzy optimization method for MOSP problem (1) with nonlinear membership function gives equivalent nonlinear programming problem as

$$\begin{aligned}&Maximize\ \ \lambda\\ &subject\ to\ \tanh\left[\left(\frac{U_1+L_1}{2}-WT(A)\right)\alpha_1\right]\ge\lambda\\ &\qquad\tanh\left[\left(\frac{U_2+L_2}{2}-\delta(A)\right)\alpha_2\right]\ge\lambda\\ &\qquad\sigma(A)\le[\sigma]\\ &\qquad A_{\min}\le A\le A_{\max},\lambda\in[0,1].\end{aligned}\tag{7}$$

Putting $\tanh^{-1}(2\lambda-1)=X$

Problem (7) is converted to

$$\begin{aligned}
&\textit{Maximize } X \\
&\textit{subject to } \alpha_1 WT(A) + X \le \alpha_1\left(\frac{U_1+L_1}{2}\right) \\
&\qquad \alpha_2 \delta(A) + X \le \alpha_2\left(\frac{U_2+L_2}{2}\right) \\
&\qquad \sigma(A) \le [\sigma] \\
&\qquad A_{\min} \le A \le A_{\max},\ X \ge 0,\ \textit{where} \tanh^{-1}(2\lambda - 1) = X.
\end{aligned} \tag{8}$$

VI. Numerical Solution of a Multi-objective Three Bar Truss Optimization Model

A well-known three bar [11] planar truss structure is considered. The design objective is to minimize weight of the structural $WT(A_1, A_2)$ and minimize the deflection $\delta(A_1, A_2)$ along u and v at loading point of a statistically loaded planar truss subjected to stress $\sigma_i(A_1, A_2)$ constraints on each of the truss members $i = 1,2,3$.

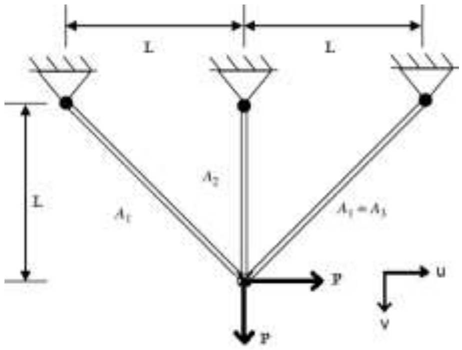

Figure 3: Design of the three-bar planar truss

The multi-objective optimization problem can be stated as follows:

$$\begin{aligned}
&\textit{Minimize } WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right) \\
&\textit{minimize } \delta_u(A_1, A_2) = \frac{\sqrt{2}PL}{EA_1} \\
&\textit{minimize } \delta_v(A_1, A_2) = \frac{\sqrt{2}PL}{E\left(A_1 + \sqrt{2}A_2\right)} \\
&\textit{subject to } \sigma_1(A_1, A_2) \equiv \frac{P(2A_1 + A_2)}{2A_1A_2 + 2A_1^2} \le \left[\sigma_1^T\right] \\
&\qquad \sigma_2(A_1, A_2) \equiv \frac{P}{\left(A_1 + \sqrt{2}A_2\right)} \le \left[\sigma_2^T\right] \\
&\qquad \sigma_3(A_1, A_2) \equiv \frac{PA_2}{2A_1A_2 + 2A_1^2} \le \left[\sigma_3^C\right] \\
&\qquad A_i^{\min} \le A_i \le A_i^{\max};\ i = 1,2
\end{aligned} \tag{9}$$

The input data for MOSOP (9) is given as follows

Table 1: Input data for crisp model (9)

Applied load P (KN)	**Volume density ρ** (KN/m^3)	**Length L** (m)	**Maximum allowable tensile stress** $\left[\sigma_1^T\right]$	**Maximum allowable tensile stress** $\left[\sigma_2^T\right]$	**Maximum allowable compressive stress** $\left[\sigma_3^C\right]$	**Young's modulus E** (KN/m^2)	$A_i^{\min}$ **and** $A_i^{\max}$ **of cross section of bars** $(10^{-4} m^2)$

			$\left(KN/m^2\right)$	$\left(KN/m^2\right)$	$\left(KN/m^2\right)$		
20	100	1	20	20	15	2×10^8	$A_1^{\min}=0.1$ $A_1^{\max}=5$ $A_2^{\min}=0.1$ $A_2^{\max}=5$

Solution: According to step 2 pay off matrix is formulated as follows:

$$\begin{array}{c} \\ A^1 \\ A^2 \\ A^3 \end{array} \begin{array}{ccc} WT(A_1,A_2) & \delta_u(A_1,A_2) & \delta_v(A_1,A_2) \\ \left(\begin{matrix} 3.732051 & 1.267949 & 0.7320508 \\ 15.37659 & 0.2828427 & 0.2096440 \\ 19.14214 & 0.2828427 & 0.1171573 \end{matrix}\right) \end{array}$$

Here $U_1=19.14214$, $L_1=3.732051$, $U_2=1.267949$, $L_2=0.2828427$, $U_3=0.7320508$, $L_3=0.1171573$.

If linear membership function is employed, the crisp model can be represented as follows

$$\begin{aligned}
&\textit{Maximize } \lambda \\
&\textit{subject to} \\
&\rho L\left(2\sqrt{2}A_1+A_2\right)+\lambda\left(U_1-L_1\right)\le U_1 \\
&\frac{\sqrt{2}PL}{EA_1}+\lambda\left(U_2-L_2\right)\le U_2 \\
&\frac{\sqrt{2}PL}{E\left(A_1+\sqrt{2}A_2\right)}+\lambda\left(U_3-L_3\right)\le U_3 \\
&\frac{P\left(2A_1+A_2\right)}{2A_1A_2+2A_1^2}\le\left[\sigma_1^T\right] \\
&\frac{P}{\left(A_1+\sqrt{2}A_2\right)}\le\left[\sigma_2^T\right] \\
&\frac{PA_2}{2A_1A_2+2A_1^2}\le\left[\sigma_3^C\right] \\
&A_i^{\min}\le A_i\le A_i^{\max};\ i=1,2,\lambda\in[0,1];
\end{aligned}$$

If we use hyperbolic membership functions, an equivalent crisp model can be formulated as follows

Maximize X
subject to

$$\alpha_1\left(\rho L\left(2\sqrt{2}A_1 + A_2\right)\right) + X \le \alpha_1\left(\frac{U_1 + L_1}{2}\right)$$

$$\alpha_2\left(\frac{\sqrt{2}PL}{EA_1}\right) + X \le \alpha_2\left(\frac{U_2 + L_2}{2}\right)$$

$$\alpha_3\left(\frac{\sqrt{2}PL}{E\left(A_1 + \sqrt{2}A_2\right)}\right) + X \le \alpha_3\left(\frac{U_3 + L_3}{2}\right)$$

$$\frac{P\left(2A_1 + A_2\right)}{2A_1A_2 + 2A_1^2} \le \left[\sigma_1^T\right]$$

$$\frac{P}{\left(A_1 + \sqrt{2}A_2\right)} \le \left[\sigma_2^T\right]$$

$$\frac{PA_2}{2A_1A_2 + 2A_1^2} \le \left[\sigma_3^C\right]$$

$$A_i^{\min} \le A_i \le A_i^{\max};\ i = 1, 2,\ X \ge 0;$$

Where $\tanh^{-1}(2\lambda - 1) = X$ and $\alpha_r = \dfrac{6}{U_r - L_r}$, $r = 1, 2, 3.$

The optimal results of model (9) using different membership function is shown in table 2

Table 2: Optimal results for Structural Weight and Deflection

Membership Function	$A_1^* \times 10^{-4} m^2$	$A_2^* \times 10^{-4} m^2$	$WT^* \times 10^2 KN$	$\delta_u^* \times 10^{-7} m$	$\delta_v^* \times 10^{-7} m$	λ
Linear	2.425445	1.568392	8.428587	0.5830738	0.3045585	0.6952
Hyperbolic	2.425445	1.568392	8.428587	0.5830738	0.3045585	0.9123

VII. CONCLUSION

In this paper, two special types of membership functions are used to solve the MOSOP. From the result, it is observed that the fuzzy optimal values does not depend on the chosen membership function whether linear or non-linear membership function is used.

Conflict of interests: The authors declare that there is no conflict of interests.

ACKNOWLEDGMENT

The authors are indebted to the referees for their valuable suggestions and careful corrections and observations.

REFERENCES

[1] Dede T,Bekirog .lu S,Ayvaz Y. “Weight minimization of trusses with genetic algorithm”. Appl Soft Comput ,11(2),2011,2pp.565–2575.

[2] Dey.Samir. and Roy,Tapan.Kumar., “ A Fuzzy Programming Technique for Solving Multi-objective Structural Problem”, International Journal of Engineering and Manufacturing” (Accepted),2014.

[3] Hans Eschenauer, Juhani Koski, and Andrzej Osyczka, editors. Multicriteria Design Optimization.Procedures and Applications. Springer-Verlag, 1990.

[4] Kaveh A, Talatahari S. “An improved ant colony optimization for the design of planar steel frames”. Eng Struct, 32(3),2010,pp-864–873.

[5] Kaveh, A., Khayatazad, M., "Ray optimization for size and shape optimization of truss structures". Computers and Structures ,11,2013,pp.82–94.

[6] Luh GC, Lin CY. "Optimal design of truss-structures using particle swarm optimization". Computers and Structures , 89(2324),2011,pp.2221–2232.

[7] R.E. Bellman and L.A. Zadeh, "Decision-making in a fuzzy environment", Management Science, 17(4),1970,pp.141-164.

[8] Rao,S.S., "Multi-objective optimization in structural design with uncertain parameters and stochastic processes". AIAA Journal,22(11),1984,pp.1670-1678.

[9] Rao,S.S., "Multi-objective optimization of fuzzy structural systems". International Journal for Numerical Methods in Engineering,24(1) ,1987,pp.1157-1171.

[10] Rao, S.S., " Description and optimum Design of Fuzzy Mathematical Systems",
Journal of Mechanisms, Transmissions, and Automation in Design, Vol.109,1987,pp.126-132.

[11] Shih,C.J., Chi,C.C. and Hsiao,J.H. "Alternative α -level-cuts methods for optimum structural design with fuzzy resources", Computers and Structures, 81,2003,pp.2579–2587.

[12] Shih,C. J. and Lee, H. W. "Level-cut Approaches of First and Second Kind for Unique Solution Design in Fuzzy EngineeringOptimization Problems", Tamkang Journal of Science and Engineering, Vol. 7, No 3,2004, pp. 189-198 .

[13] Wang,G.Y.,Wang, W.Q., " Fuzzy optimum design of structure." Engineering Optimization,8,1985,pp.291-300.

[14] Xu, C. "Fuzzy optimization of structures by the two-phase method",Computer and Structure, 31(4),1989,pp.575–580.

[15] Yeh, Y.C, and Hsu, D.S. "Structural optimization with fuzzy parameters".Computer and Structure, 37(6), 1990,pp.917–924,

[16] Zadeh,L. A., Fuzzy set, Information and Control, 8(3),1965,pp-338-353.

[17] Zimmermann, H.J., "fuzzy linear programming with several objective function" Fuzzy sets and systems,1,1978,pp.46-55.