Boundary-Layer Flow over a Porous Medium of a Nanofluid Past from a Vertical Cone

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Abstract— **This work is focused on the study of the natural convection boundary-layer flow over a downward-pointing vertical cone in a porous medium saturated with a non-Newtonian nanofluid in the presence of heat generation or absorption. The boundary layer equations were normalized via similarity variables and solved numerically. The generalized governing equations derived in this work can be applied and parametric study of the physical parameters is conducted. These equations are solved by using Raunge kutta. Comparison of numerical results with previously published work, are performed and good agreement is obtained.**

Keywords- Non-Newtonian, Nano fluid, Heat generation or absorption, Porous media, Similarity solution.

I. INTRODUCTION

The natural convection flow over a surface embedded in saturated porous media is encountered in many engineering problems such as the design of pebble-bed nuclear reactors, ceramic processing, crude oil drilling, geothermal energy conversion, use of fibrous material in the thermal insulation of buildings, catalytic reactors and compact heat exchangers, heat transfer from storage of agricultural products which generate heat as a result of metabolism, petroleum reservoirs, storage of nuclear wastes, etc. The derivation of the empirical equations which govern the flow and heat transfer in a porous medium has been discussed in [1-5]. The natural convection on vertical surfaces in porous media has been studied used Darcy's law by a number of authors [6–20]. Boundary layer analysis of natural convection over a cone has been investigated by Yih [21-24]. Murthy and Singh [25] obtained the similarity solution for non-Darcy mixed convection about an isothermal vertical cone with fixed apex half angle, pointing downwards in a fluid saturated porous medium with uniform free stream velocity, but a semi-similar solution of an unsteady mixed convection flow over a rotating cone in a rotating viscous fluid has been obtained Roy and Anilkumar [26]. The laminar steady no similar natural convection flow of gases over an isothermal vertical cone has been investigated by Takhar et al. [27]. The development of unsteady mixed convection flow of an incompressible laminar viscous fluid over a vertical cone has been investigated by Singh and Roy [28] when the fluid in the changed from its ambient temperature. An analysis has been carried out by external stream is set into motion impulsively, and at the same time the surface temperature is suddenly Kumari and Nath [29] to study the non-Darcy natural convention flow of Newtonian fluids on a vertical cone embedded in a saturated porous medium with power-law variation of the wall temperature/concentration or heat/mass flux and suction/injection. Cheng [30-34] focused on the problem of natural convection from a vertical cone in a porous medium with mixed thermal boundary conditions, Soret and Dufour effects and with variable viscosity. Gorla and Chamkha [35] studied the natural convective boundary layer flow over a non-isothermal vertical plate embedded in a porous medium saturated with a Nano fluid. Khan and Pop [36] examined the free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with a nanofluid. Chamkha et al. [37] presented the non-similar solutions for natural convective boundary layer flow over a sphere embedded in a porous medium saturated with a nanofluid. Improving the technology, limit in enhancing the performance of conventional heat transfer is a main issue owing to low thermal conductivity of the most common fluids such as water, oil, and ethylene- glycol mixture. Since the thermal conductivity of solids is often higher than that of liquids, the idea of adding particles to a conventional fluid to enhance its heat transfer characteristics was emerged. Owing to wide range of technological applications including electronic systems, hydrocyclone devices, quenching and nuclear reactors, flow over a sphere saturated in nanofluid has attracted so many attention. In this paper, the basic boundary layer equations have been reduced to a two-point boundary value problem via similarity variables, and solved numerically.

II. DESCRIPTION OF PROBLEM AND FORMULATION

The problem of natural convection about a downward-pointing vertical cone of half angle ¥ embedded in a porous medium saturated with a non-Newtonian power-law nanofluid. The origin of the coordinate system is placed at the vertex of the full cone, with x being the coordinate along the surface of the cone measured from the origin and y being the coordinate perpendicular to the conical surface Fig (1). The temperature of the porous medium on the surface of the cone is kept at constant temperature T_w , and the ambient porous medium temperature is held at constant temperature T_∞ .The nanofluid properties are assumed to be constant except for density variations in the buoyancy force term. The thermo physical properties of the nanofluid are given in Table 1 (see Oztop and Abu-Nada [39]). Assuming that the thermal boundary layer is sufficiently thin compared with the local radius, the equations governing the problem of Darcy flow through a homogeneous porous medium saturated with power-law nanofluid near the vertical cone can be written in two-dimensional Cartesian coordinates (x,y). In this analysis we will consider the volumetric rate of heat generation, $Q\left(W/m^3\right)$as:

$$Q = \begin{cases} Q(T - T_\infty) & T \geq T_\infty \\ 0 & T < T_\infty \end{cases}$$

where Q is the heat generation coefficient.

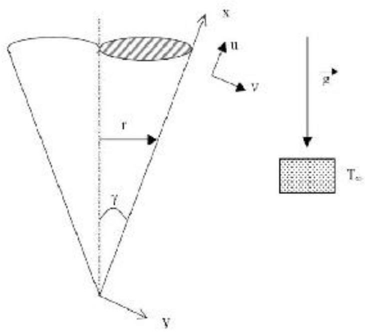


Figure 1. A schematic diagram of the physical model.

The velocity and temperature fields in the boundary layer are governed by the two-dimensional boundary layer equations for mass, momentum and thermal energy:

Continuity equation:

$$\frac{\partial(r^m u)}{\partial x} + \frac{\partial(r^m v)}{\partial y} = 0 \qquad (1)$$

Momentum equation:

$$\frac{\partial u^n}{\partial y} = \frac{(\rho\beta)_{nf}}{\mu_{nf}} kg \cos\gamma \frac{\partial T}{\partial y} \qquad (2)$$

Integration the momentum Eq. (2) we have:

$$\frac{\mu_{nf}}{\mu_f} u^n = \frac{(\rho\beta)_{nf}}{\mu_f} kg \cos\gamma (T - T_\infty) \qquad (3)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{(\rho c_p)_{nf}}(T-T_\infty) \tag{4}$$

Where u and v are the volume-averaged velocity components in the x and y directions, respectively, T is the volume-averaged temperature. n is the power-law viscosity index of the power-law nanofluid and g is the gravitational acceleration. $m =$ ¥ $= 0$ corresponds to flow over a vertical flat plate and $m = 1$ corresponds to flow over a vertical cone. n is the viscosity index. For the case of $n = 1$, the base fluid is Newtonian. We note that $n < 1$ and $n > 1$ represent pseudo-plastic fluid and dilatants fluid, respectively. Property °$_{nf}$ and «$_{nf}$ are the density and effective viscosity of the nanofluid, and K is the modified permeability of the porous medium. And Q is the heat generation coefficient. Furthermore, ~$_{nf}$ and $_{nf}$ are the equivalent thermal diffusivity and the thermal expansion coefficient of the saturated porous medium, which are defined as (see Khanafer et al. [36]):

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$$

$$(°C_p)_{nf}=(1-¤)(°C_p)_f+¤(°C_p)_s$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\emptyset(k_f-k_s)}{(k_s+2k_f)-\emptyset(k_f-k_s)} \tag{5}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$$

Here ¤ is the solid volume fraction. The associated boundary conditions of Eqs. (1)-(4) can be written as:

$$v=0;\quad T=T_w \quad at \quad y=0;$$
$$u=0;\quad T\to T_\infty \quad as \quad y\to\infty; \tag{6}$$

where «$_f$ is the viscosity of the basic fluid, °$_f$ and °$_s$ are the densities of the pure fluid and nanoparticle, respectively, $(°C_p)_f$ and $(°C_p)_s$ are the specific heat parameters of the base fluid and nanoparticle, respectively, k_f and k_s are the thermal conductivities of the base fluid and nanoparticle, respectively. The local radius to a point in the boundary layer r can be represented by the local radius of the vertical cone r=x sinγ.

TABLE 1 Thermo-physical properties of water and nanoparticles

Physical properties	**Pure water**	**Cu**	**Al_2O_3**	**TiO_2**
C_p (J/kg K)	4179.000	385	765	686.2000
° (Kg/m³)	997.100	8933	3970	4250.000
K (W/mK)	0.613	400	40	8.9538

By introducing the following non-dimensional variables:

$$\eta = \frac{y}{x} Ra_x^{1/2} \qquad f(\eta) = \frac{\psi(x,y)}{a_f r^m Ra_x^{1/2}}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

The continuity equation is automatically satisfied by defining a stream function · (x, y) such that:

$$r^m u = \frac{\partial \psi}{\partial y} \quad and \quad r^m v = \frac{\partial \psi}{\partial x} \tag{8}$$

where;

$$Ra_x = \left(\frac{x}{\sim_f}\right)\left[\frac{kg(\circ\ )_f \cos ¥ \left(T_w - T_\infty\right)}{«_f}\right]^{1/n} \tag{9}$$

Substituting variables (7) into Eqs. (1)–(6), we obtain the following system of ordinary differential equations:

$$\frac{1}{(1-\emptyset)^{2.5}}(f')^n = \left[1 - \emptyset + \emptyset\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\theta \tag{10}$$

$$\frac{k_{nf}}{k_f}\theta'' + \left(m + \frac{1}{2}\right)\left[1 - \emptyset + \emptyset\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]f\theta' + \lambda\theta = 0 \tag{11}$$

$$\lambda = \frac{Q v_f x}{\cos\gamma (T_W - T_\infty)\, kg(\rho\beta c_p)_f} \tag{12}$$

along with the boundary conditions:

$$f(0) = 0 \quad , \quad \theta(0) = 0$$
$$f'(\infty) = 0 \quad , \quad \theta(\infty) = 1 \tag{13}$$

where primes denote differentiation with respect to ¦, the quantity of practical interest, in this chapter is the Nusselt number Nu_x which is defined in the form. and ª is the heat generation or absorption parameter.

$$Nu_x = \frac{h_x}{k_m} = \frac{-\frac{\partial T}{\partial y}|y = 0^x}{T_w - T_\infty} = -Ra_x^{\frac{1}{2}}\theta'(0) \tag{14}$$

where h denotes the local heat transfer coefficient.

III. Result and Discussion

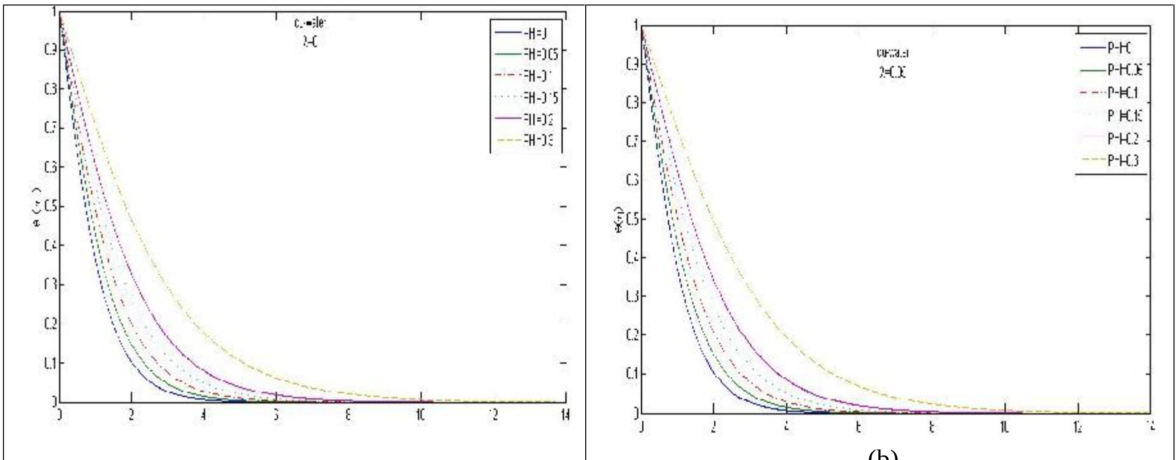
In this study we have presented similarity reductions for the effect of nanoparticle volume transformations. The numerical solutions of the resulted similarity reductions are obtained for the original variables which are shown in Eqs. (10) and (11) along with the boundary conditions (13) by using the secant method. The physical quantity of interest here is the Nusselt number Nu_x and it is obtained and shown in Eqs. (14) The distributions of the velocity $f'(¦)$, the temperature ¯(¦) from Eqs.(10) and (11) and the Nusselt number in the case of Cu-water and Ag-water are shown in Figs. 2–8. The computations are carried for various values of the nanoparticles volume fraction for different types of nanoparticles, when the base fluid is water. Nanoparticles volume fraction ¤ . is varied from 0 to 0.3. The nanoparticles used in the study are from Copper (Cu), Silver (Ag), Alumina (Al_2O_3) and Titanium oxide (TiO_2).

Table 2: Values of $-\theta'(0)$ for various ϕ and λ when n=1.

λ	ϕ	Cu	Ag	Al_2O_3	TiO_2
0.0	0	0.7688	0.7688	0.7688	0.7688
	0.05	0.6626	0.6626	0.6535	0.6598
	0.1	0.5705	0.5705	0.5545	0.5649
	0.15	0.4903	0.4901	0.4689	0.4820
	0.2	0.4199	0.4197	0.3949	0.4094
	0.25	0.3581	0.3578	0.3306	0.3458
	0.3	0.3036	0.3032	0.2749	0.2902

λ	ϕ	Cu	Ag	Al_2O_3	TiO_2
0.05	0	0.7394	0.7394	0.7394	0.7394
	0.05	0.6331	0.6328	0.6235	0.6295
	0.1	0.5407	0.5402	0.5234	0.4487
	0.15	0.4599	0.4592	0.4366	0.4487
	0.2	0.3888	0.3879	0.3609	0.3742
	0.25	0.3260	0.3249	0.2947	0.3082
	0.3	0.2703	0.2691	0.2366	0.2496

λ	ϕ	Cu	Ag	Al_2O_3	TiO_2
0.1	0	0.7093	0.7093	0.7093	0.7093
	0.05	0.6027	0.6027	0.5924	0.5982
	0.1	0.5098	0.5098	0.4912	0.5004
	0.15	0.4283	0.4281	0.4027	0.4139
	0.2	0.3561	0.3559	0.3250	0.3368
	0.25	0.2919	0.2916	0.2561	0.2677
	0.3	0.2344	0.2339	0.1944	0.2050

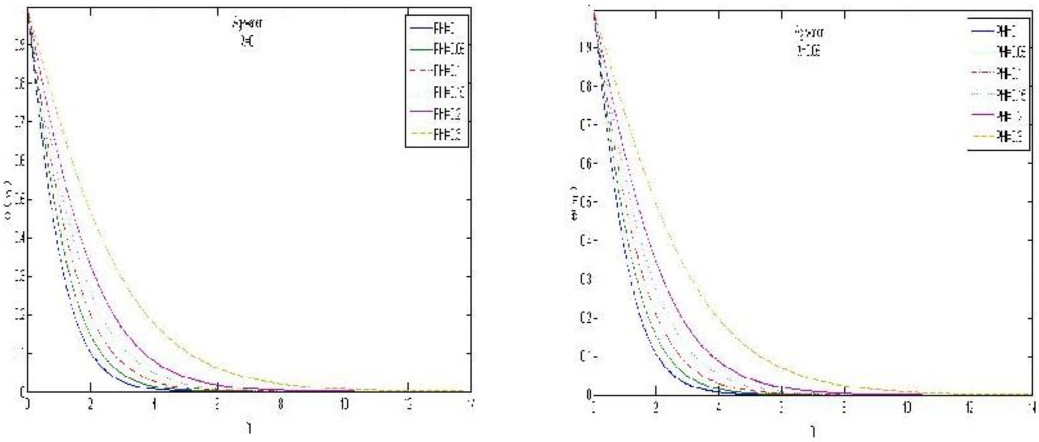


(a) (b)

Figure 2: Temperature profiles at different volume fraction of nanoparticles (Cu - water): (a)with the production of heat,; λ = 0.05 (b) with out generating heat.

Figure 2 Effect of heat production and changes in the volume fraction of nanoparticles shows the temperature distribution. Nanofluids made of copper particles and the fluid is water. With increasing volume fraction of nanoparticles from 0 to 0.3 in both cases, i.e. no heat generation and temperature distribution, heat production increases. Large impact on the temperature profile does not generate heat and temperature distribution with no heat production is almost identical.

Figure 3 Effect of heat production and changes in the volume fraction of nanoparticles shows the temperature distribution. The nanoparticles are made of aluminum and liquid water. The nano-fluid temperature distribution of copper nano-fluids - water is almost identical. With increasing volume fraction of nanoparticles in both cases, i.e. no heat generation and temperature distribution, heat production increases. The heat does not have a large impact on the temperature profile.

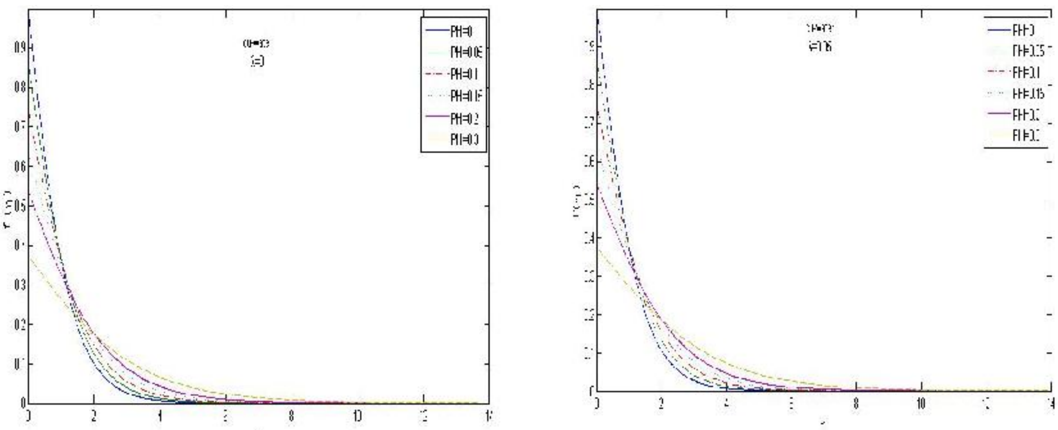


(a) (b)

Figure 3: Temperature profiles at different volume fraction of nanoparticles (alumina - water): (a) the production of heat λ = 0.05; (B) without generating heat.

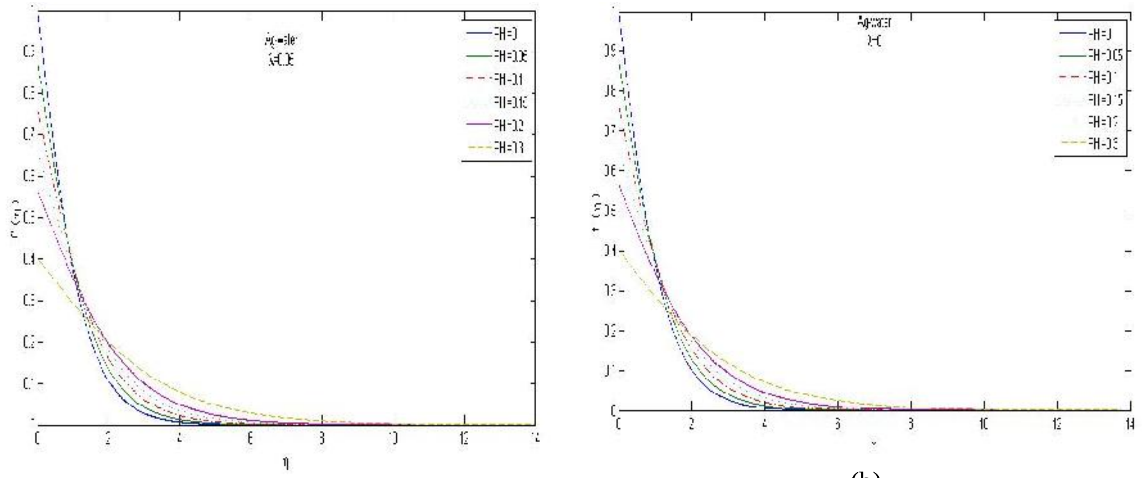
Figure 4 shows the effect of changing temperature and volume fraction of nanoparticles on the velocity distribution show. Nanofluids made of copper particles and the fluid is water. Nanvaz particle volume fraction increases from 0 to 0.3 in both cases, ie no heat production and heat production. Velocity distribution in the 1.8> η decreases in η> 1.8 increases. Heat generating has not big effects on the velocity profile and it is almost same as another case (No Heat production case).

Figure 5 shows the effect of changing temperature and volume fraction of nanoparticles on the velocity distribution show. The nanoparticles are made of aluminum and liquid water. The nano-fluid flow rate nm copper - water is higher, but the difference in speed is very low. With increasing volume fraction of nanoparticles in both cases, i.e. no heat production and heat production, velocity distribution in the 1.8> η decreases in η> 1.8 increases. Big effect on the velocity profile does not generate heat and produce almost no heat is the same.



(a) (b)

Figure 4: The velocity profiles at different volume fraction of nanoparticles (Cu - water): (a) the production of heat,; λ = 0.05 (b) without generating heat.



(a) (b)

Figure 5: The velocity profiles at different volume fraction of nano particles (aluminum - water): (a) the production of heat,; λ = 0.05 (b) without generating heat.

IV. CONCLUSIONS

A numerical study on the steady boundary layer flow and heat transfer of nanofluid over a downward-pointing vertical cone in a porous medium saturated with a non-Newtonian nanofluid in the presence of heat generation or absorption has been performed. Boundary layer equations were solved numerically by the Rung Kutta method with shooting technique on the natural convection boundary-layer flow. The generalized governing equations derived in this work can be applied and parametric study of the physical parameters is conducted. These equations are solved by using Raunge kutta. Results show that, with increasing volume fraction of nanoparticles from 0 to 0.3 in both cases, i.e. no heat generation and temperature distribution, heat production increases. Also, It is shown that, With increasing volume fraction of nanoparticles in both cases, heat production increases. It is seen that, increasing volume fraction of nanoparticles in both cases, velocity distribution in the 1.8> η decreases in η> 1.8 increases. Comparisons with previously published work are performed and good agreement is obtained.

REFERENCES

[1] Cheng P (1978) Heat transferin geothermal systems. Adv. Heat Transfer14: 1–105.

[2] Nield DA, Bejan A (1999) Convection in Porous Media. second ed.,Springer: New York.

[3] Vafai K (2000) Handbook of Porous Media. Marcel Dekker: New York.

[4] Pop I, Ingham DB (2001) Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media. Pergamon Press:Oxford.

[5] Ingham DB, Pop I (2002) Transport Phenomena in Porous Media. Pergamon Press: Oxford

[6] Cheng P, Minkowycz WJ (1977) Free convectionabout a vertical flat plate embedded in a porous medium with application to heat transfer from a dike. J. Geophys. Res.82(14): 2040–2044

[7] Gorla RSR, Tornabene R (1988) Free convectionfrom a vertical plate with nonuniform surface heat flux and embedded in a porous medium. Transp. Porous Media3 : 95–106.

[8] Bakier AY, Mansour MA, Gorla RSR, EbianaAB (1997) Nonsimilar solutions for free convection from a vertical plate in porous media. Heat Mass Transfer33: 145–148.

[9] Gorla RSR, Mansour MA, Abdel-Gaied SM (1999) Natural convection from a vertical plate in a porous medium using Brinkman’s model. Transp. Porous Media36 : 357–371.

[10] Mulolani I, Rahman M (2000) Similarity analysis for natural convection from a vertical plate with distributed wall concentration. Int. J. Math. math. Sci.23(5) : 319–334.

[11] Jumah RY, Mujumdar AS (2000) Free convection heat and mass transfer of nonNewtonian power law fluids with yield stress from a vertical plate in saturated porous media.Int. Comm. Heat Mass Transfer27 : 485–494.

[12] Groşan T, Pop I (2001) Free convection over a vertical flat plate with a variable wall temperature and internal heat generation in a porous medium saturated with a nonNewtonian fluid. Technische Mechanik21(4) : 313–318.

[13] Kumaran V, Pop I (2006) Steady free convection boundary layer over a vertical flat plateo embedded in a porous medium filled with water at 4 C. Int. J. Heat Mass Transfer49 : 3240–3252.

[14] Magyari E, Pop I, Keller B (2006) Unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium. Transp. Porous Media62 : 233–249.

[15] Cheng C-Y (2006) Natural convection heatand mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes. Int. Comm. Heat Mass Transfer33 : 1156–1164.

[16] Chamkha AJ, Al-Mudhaf AF, Pop I (2006) Effect of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium. Int. Comm. Heat Mass Transfer33 : 1096–1102.

[17] Magyari E, Pop I, Postelnicu A (2007) Effect of the source term on steady free convection boundary layer flows over an vertical plate in a porous medium. Part I. Transp. Porous Media67 : 49–67.

[18] Nield DA, Kuznetsov AV (2008) Natural convection about a vertical plate embedded in a bidisperse porous medium. Int. J. Heat Mass Transfer51 : 1658–1664.

[19] Mahdy A, Hady FM (2009) Effect of thermophoretic particle deposition in nonNewtonian free convection flow over a vertical plate with magnetic field effect. J. NonNewtonian Fluid Mech.161: 37–41.

[20] Ibrahim FS, Hady FM, Abdel-Gaied SM, Eid MR (2010) Influence of chemical reaction on heat and mass transfer of non-Newtonian fluid with yield stress by free convection from vertical surface in porous medium considering Soret effect. Appl. Math. Mech. -Engl. Ed.31(6): 675–684.

[21] Yih KA (1997) The effect of uniform lateral mass flux effect on free convection about a vertical cone embedded in a saturated porous medium. Int. Comm. Heat Mass Transfer 24(8): 1195–1205.

[22] Yih KA (1998) Uniform lateral mass flux effecton natural convection of non-Newtonian fluids over a cone in porous media.Int. Comm. Heat Mass Transfer25(7): 959–968.

[23] Yih KA (1999) Effect of radiation on natural convection about a truncated cone. Int. J. Heat Mass Transfer42: 4299–4305.

[24] Yih KA (1999) Coupled heat and mass transfer by free convection over a truncated cone in porous media: VWT/VWC or VHF/VMF. Acta Mech.137 : 83–97.

[25] Murthy PVSN, Singh P (2000) Thermal dispersion effects on non-Darcy convection over a cone. Compu. Math. Applications40: 1433–1444.

[26] Roy S, Anilkumar D (2004) Unsteady mixed convection from a rotating cone in a rotating fluid due to the combined effects of thermal and mass diffusion. Int. J. Heat Mass Transfer47:1673–1684.

[27] Takhar HS, Chamkha AJ, Nath G (2004) Effect of thermophysical quantities on the natural convection flow of gases over a vertical cone. Int. J. Eng. Sci.42: 243–256.

[28] Singh PJ, Roy S (2007) Unsteady mixed convection flow over a vertical cone due to impulsive motion. Int. J. Heat Mass Transfer50: 949–959.

[29] Kumari M, Nath G (2009) Natural convection from a vertical cone in a porous medium due to the combined effects of heat and mass diffusion with non-uniform wall temperature/concentration or heat/mass flux and suction/injection. Int. J. Heat Mass Transfer52: 3064–3069.

[30] Cheng C-Y (2009) Natural convection heat transfer of non-Newtonian fluids in porous media from a vertical cone under mixed thermal boundary conditions. Int. Comm. Heat Mass Transfer36: 693–697.

[31] Cheng C-Y (2009) Soret and Dufour effects on natural convection heat and mass transfer from a vertical cone in a porous medium, Int. Comm. Heat Mass Transfer36 : 1020–1024.

[32] Cheng C-Y (2010) Soret and Dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration. Int. Comm. Heat Mass Transfer37 : 1031–1035.

[33] Cheng C-Y (2010) Soret and Dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration. Int. Comm. Heat Mass Transfer37 : 1031–1035.

[34] Cheng C-Y (2010) Nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium withvariable viscosity. Int. Comm.

[35] R.S.R. Gorla, A. Chamkha, Natural convective boundary layer flow over a nonisothermal vertical plate embedded in a porous medium saturated with a nanofluid, Nanoscale and Microscale Thermophysical Engineering 15 (2011) 81–94.

[36] W.A. Khan, I. Pop, Free convection boundary layerflow past a horizontalflat plate embedded in a porous mediumfilled with a nanofluid, ASME Journal of Heat Transfer 133 (2011) 094501.1–094501.4.

[37] A. Chamkha, R.S.R. Gorla, K. Ghodeswar, Non-similar solution for natural convective boundary layerflow over a sphere embedded in a porous medium saturated with a nanofluid, Transport in Porous Media 86 (2011) 13–22