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Nanofluid Effects on the Flow with Suction or Injection Over a Moving Surface and Heat Transfer in Boundary Layer

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Abstract— **In this study effect of boundary layer flow over flat plate in a nanofluid with two dimensional boundary layer. Uniform suction and injection on the flow field and heat transfer is analyzed. The nonlinear partial differential equation reduced by similarity transformation to the non-coupled nonlinear ordinary differential equations. Numerical shooting technique with a fourth-order Runge-Kutta scheme had used to obtain the solution of the boundary value problem. Using different types of nanoparticles as Cu (cuprum), Al_2O_3 (aluminum) and TiO_2 (titanium).The result of injection accelerates boundary layer separation.**
Keywords-Nanofluid; Boundary layer injection; suction; Heat transfer.

I. Introduction

Small solid particles in fluids improving the heat transfer of fluids .this new kind of fluids named nanofluid and was introduced by choi in 1995 [1]. nanofluid consists mixture of liquid like water and solid nanoparticles with high conductivity and low volume fraction. Enormous heat transfer can be results. This Nano fluid usually are very stable and without additional problems like erosion, pressure drop and non-Newtonian behavior. The Argonne National Laboratory has studied to use nanoparticles around a decade ago [2, 3]. Nanoparticles can change heat transfer and thermal properties. Therefore, it is very useful to enhance heat transfer in separated regions by using nanofluids. Such enhancement is accomplished by increasing the value of convective heat transfer coefficient (or Nusselt number) in separated flows. Suction or injection has affected the heat transfer rate at the surface. For example suction help to increase the skin fraction and heat transfer coefficients but injection tends to decrease that .[4]Injection of fluid through a porous bounding heated or cooled surface is usually interest in practical application including film cooling, control of boundary layer etc. This can lead to enhanced heating or cooling of the system and can help to delay the transition from laminar flow [5].

II. Description of Problem and formulation

Consider the boundary layer flow of a viscous and incompressible nanofluid on a moving semi-infinite permeable flat plate. nanofluids based is water and nanoparticles are different types of Cu, Al_2O_3 and TiO_2. The flow is assumed laminar with no slip between them. The plate moves with a constant velocity $\bar{u}_w = \text{a}U$, where a is a constant and U is the constant free stream velocity or the velocity of the far field (inviscid) flow [6]. Coordinate system is considerate Cartesian$(\bar{x}, \bar{y})$, where $\bar{x}$ and $\bar{y}$ are the coordinates measured along the plate and normal to it. When $\bar{y} \geq 0$ flow takes place and temperature of moving plate is constant T_w, ambient nanofluid temperature is T_∞ and we have heat transfer at $T_w > T_\infty$. Using the nanofluid model proposed by Tiwari and Das [3]. The basic steady conservation of mass, momentum and energy equations are fallowing:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{1}{\circ_{nf}}\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{\ll_{nf}}{\circ_{nf}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{2}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}=-\frac{1}{\circ_{nf}}\frac{\partial \bar{p}}{\partial \bar{y}}+\frac{\ll_{nf}}{\circ_{nf}}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\sim_{nf}\left(\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}\right) \tag{4}$$

The boundary condition considerate following:

$$\bar{v}=\bar{v}_w,\bar{u}=\bar{u}_w=ªU,T=T_w,\text{ at } \bar{y}=0 \tag{5}$$
$$\bar{u}=\bar{u}_e=U,\bar{v}=0,T=T_\infty,\quad \bar{p}=p_\infty,\text{ as } y\to\infty$$

Here $\bar{u}$ and $\bar{v}$ are the velocity components along $\bar{x}$ and $\bar{y}$ axes. $\bar{p}$ is the fluid pressure, T is temperature of the nanofluid, $\circ_{nf}$ is density of nanofluid , $\sim_{nf}$ is thermal diffusivity of the nanofluid and $\ll_{nf}$ is the viscosity of nanofluid [7].

The properties of nanofluid are defined as follows:

$$\sim_{nf}=\frac{k_{nf}}{\left(\circ C_p\right)_{nf}}$$

$$\circ_{nf}=\left(1-\ddot{}\right)\circ_f+\ddot{}\,\circ_s, \tag{6}$$

$$\ll_{nf}=\frac{\ll_f}{\left(1-\ddot{}\right)^{2.5}},$$

$$\left(\circ C_p\right)_{nf}=\left(1-\ddot{}\right)\left(\circ C_P\right)_f+\ddot{}\left(\circ C_P\right)_s,$$

$$\frac{k_{nf}}{k_f}=\frac{\left(k_s+2k_f\right)-2\ddot{}\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\ddot{}\left(k_f-k_s\right)},$$

Here k_{nf} is thermal conductivity of nanofluid , $\left(\circ C_p\right)_{nf}$ is heat capacity of nanofluid , $\ddot{}$ is nanoparticle volume fraction , $\circ_f$ is density of fluid, $\circ_s$ is density of solid fraction , k_f and k_s are thermal conductivity of fluid and the solid fractions, $\ll_{nf}$ is viscosity of fluid and $\ll_f$ approximated by Brinkman [8]. Variables in boundary layer are follows:

$$x=\frac{\bar{x}}{L},y=\mathrm{Re}^{1/2}\left(\frac{\bar{y}}{L}\right),u=\frac{\bar{u}}{U} \tag{7}$$

$$v=\mathrm{Re}^{1/2}\left(\frac{\bar{v}}{L}\right),u_e=\frac{\bar{u}_e}{U} \tag{8}$$

$$-=\frac{T-T_\infty}{T_w-T_\infty} \tag{9}$$

$$P=\frac{\bar{P}-P_\infty}{\left(\circ_f U^2\right)},u_w=\frac{\bar{u}_w}{U} \tag{10}$$

Here L is length of the plate, Re is Reynolds number.

In this problem we assumed have no pressure gradient so dimensionless boundary layer an equation follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (11)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu_{nf}}{\rho_{nf}\nu_f}\right)\frac{\partial^2 u}{\partial y^2} \qquad (12)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{\alpha_{nf}}{\nu_f}\frac{\partial^2\theta}{\partial y^2} \qquad (13)$$

For equations (5) boundary condition as follows:

$$u=u_w=\lambda, v=v_w, \theta=1 \text{ at y=o} \qquad (14)$$
$$u=u_e=1, \theta=0, \text{ as } y\to\infty.$$

Note that for suction $v_w<0$ and $v_w=0$ for impermeable surface and for injection $v_w>0$.
We can use similar solution for equations 11-13 as follows:

$$\eta=\frac{y}{(2x)^{1/2}}, \qquad (15)$$

$$f(\eta)=\frac{\psi}{(2x)^{1/2}}, \qquad (16)$$

$$g(\eta)=\theta, \qquad (17)$$

Note: that here ψ is stream function and usually obtained from as this:

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x}, \qquad (18)$$

For different order equations 11-13 we can use this formula:

$$v_w=-\frac{f_0}{(2x)^{1/2}}, \qquad (19)$$

Here $f_0>0$ consider for suction, $f_0<0$ we have injection $f_0=0$ for impermeable surface and $f_0=f(0)$ is non-dimensional constant for which determines the transpiration rate for them .[9, 10] Using Eqs. (15)-(17) and (19) ,(12) and (13) are reduce to this ordinary differential equations:

$$\frac{f'''}{(1-\varphi)^{2.5}\left[1-\varphi+\varphi\left(\frac{\rho_s}{\rho_f}\right)\right]}+ff''=0 \qquad (20)$$

$$\frac{1}{\Pr}\left[\frac{g'' k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}\right]+fg'=0 \qquad (21)$$

Boundary conditions are as follows:

$$f(0)=f_0, f'(0)=\lambda, g(0)=1 \qquad (22)$$
$$f'(\eta)=1, g(\eta)=0, \eta\to\infty$$

The physical properties are friction coefficient and the Nusselt number, which define as:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}$$
$$Nu = \frac{Lq_w}{k_f\left(T_w - T_\infty\right)} \qquad (23)$$

Here τ_w and q_w are surface shear stress and surface heat flux, which become to:

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (24)$$

From Eqs.(7) –(10) and (15)-(16) ,formulation (24) obtained:

$$\left(2\,\mathrm{Re}_x\right)^{1/2} C_f = \frac{f''(0)}{\left(1-\phi\right)^{2.5}}, \qquad (25)$$

$$\left(2/\mathrm{Re}_x\right)^{1/2} Nu = -\frac{k_{nf}}{k_f} g'(0) \qquad (26)$$

III. RESULT AND DISCUSSION

For solve equations (20) and (21) use numerical solution and using the Runge-Kutta method with shooting technique. Result find missing slopes $f''(0), g'(0)$ for different values parameters. Three types of nanoparticles were considered such as copper Cu, alumina Al_2O_3, TiO_2 (titanium) and parameters, namely the nanoparticle volume fraction ϕ , the moving parameter λ and the suction or injection parameter f_0 considered [7]. For water the value of prandtl number pr as 6.2 .and thermo physical properties of fluid and nanoparticles as follow in TABLE 1.

TABLE 1 Thermo physical property [7].

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
C_p (J/kg K)	4179.000	385	765	686.2000
ρ (Kg/m³)	997.100	8933	3970	4250.000
K (W/mK)	0.613	400	40	8.9538

For regular Newtonian fluid $\phi = 0$ and nanoparticles as 0 to 0.2 and f_0 as -0.6 , 0 and 0.6. Figure 1 Nusselt number, eq. (26) with different values of f_0 and nanoparticle volume fraction parameter ϕ for three different values Cu, Al_2O_3 and TiO_2 . This figure shows that, with increased volume fraction parameter these quantities increase.

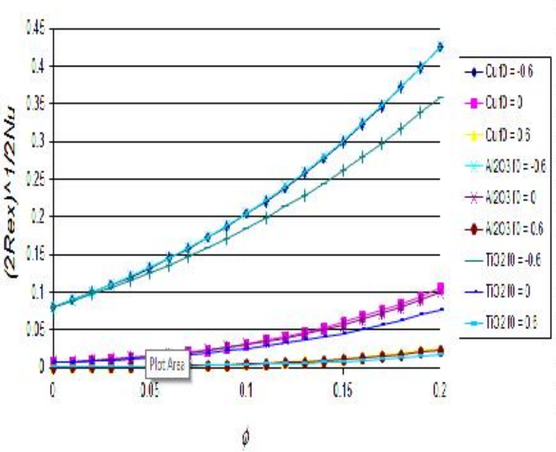

Fig.1 Variation of the local Nusselt number with ¨ for different nanoparticles and $f_0, pr = 6.2,$ ª $= 0.5$

It is seen that, TiO_2 has the lowest value of thermal conductivity and lowest heat transfer rate.

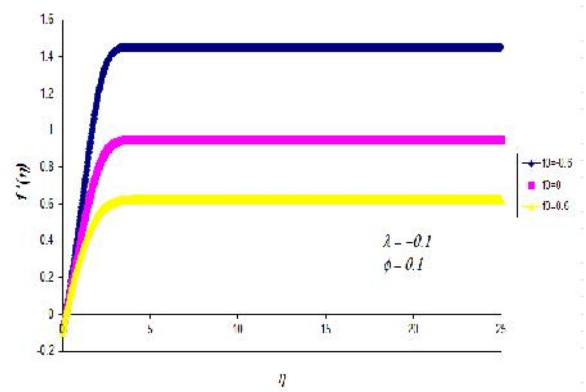

Fig.2 Velocity profiles for Cu-water nanofluid and difference values of f_0 and ª $= -1, pr = 6.2$

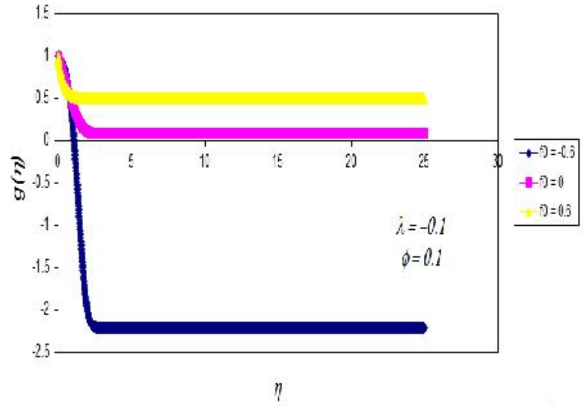

Fig.3 Temperature profiles for Cu-water nanofluid and different values of f_0 and $pr = 6.2,$ ª $= -0.1$

Figure 2 and 3 show the velocity profile for Cu-water nanofluid and different values of f_0 for Pr as 6.2 and ª $= -0.1$.

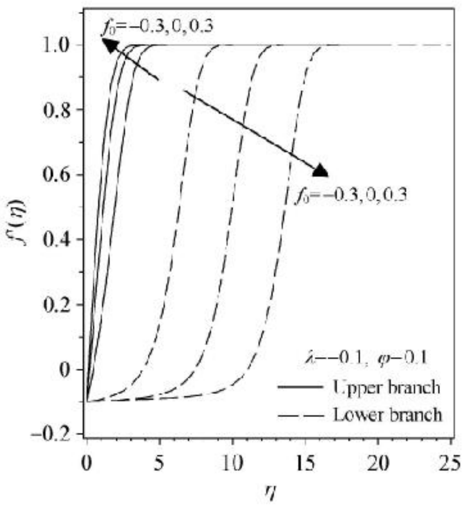

Fig.4 Velocity profiles for Cu-water nanofluid and different values of f_0 when $pr = 6.2$ and $\lambda = -0.1$ [11].

IV. Conclusions

Boundary layer equations were solved numerically by the Rung Kutta method with shooting technique. Results for injection and suction parameters f_0 and nanoparticle volume fraction φ, local Nusselt number $-g'(0)$ and moving parameter λ. Variation of the local Nusselt number with φ for different nanoparticles and variable parameters is studied. It is shown that, TiO_2 has the lowest value of thermal conductivity and lowest heat transfer rate.

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