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Economic Statistical Design of $\overline{X}$ Control Chart using Genetic Algorithm

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Abstract - **Control chart are widely used to establish and maintain statistical control of a process. In other words it is a tool used to monitor the processes and to assure that they remain "in control" or stable. The $\overline{X}$ control chart is preferred most in comparison to any other control chart technique if quality is measured on a regular scale. The design of a control chart involves the selection of the parameters like sample size (n), sampling interval (h), and control limits width (L). The design of a control chart also has an economic aspect as it involves the costs of sampling, inspection, checking for out of control signals, and cost of non-conforming units reaching the consumer. Economic-statistical design is basically a combination of economic and statistical design of control chart. In this type of design, the total cost of maintaining the control chart need to be minimized and at the same time Type-I and Type-II errors are not allowed to exceed their permissible level. In the present work, a genetic algorithm has been developed for the economic design of the $\overline{X}$ control chart (ESDCC-GA) under uniform and non-uniform sampling interval that gives the optimum values of the sample size, sampling interval and width of control limits such that the expected total cost per hour (ECT) is minimized. The results obtained are found to be better compared to that reported in the literature.**

Keywords - Control chart, economic statistical design, expected cost per hour, genetic algorithm.

I. INTRODUCTION

ISO, an international body for formulating standards, has defined quality as degree to which a set of inherent characteristics fulfils requirements. Degree refers to a level to which a product or service satisfies. So, depending upon the level of satisfaction, a product may be termed as excellent, good or poor quality product. Inherent characteristics are those features that are a part of the product and are responsible to achieve satisfaction. Requirements refer to the needs of customer, needs of organization and those of other interested parties (e.g. regulatory bodies, suppliers, employees, community and environment) or it is the expectations that may be stated, generally implied or obligatory (ISO 1802:1994).

Improving the quality of the output is a major factor for a successful and competitive business in the market. Statistical process control (SPC) is one of the best technical tools for improving product and service quality. SPC consists of methods for understanding, monitoring and improving process performance over time (Woodall, 2000). It is now realized that SPC is not just a collection of techniques, but a way of thinking about quality improvement, and it is regarded in many organizations as an important element of Total Quality Management (Caulcutt, 1995).

Control chart is one of the widely used statistical process control tools. It is used to statistically monitor the process through sampling inspection instead of 100% inspection. It only indicates whether the process is in-control or out-of-control but it cannot on its own rectify the process. It presents a graphic display of process stability or instability over time. One goal of using a control chart is to achieve and maintain process stability. Process stability is defined as a state in which a process has displayed a certain degree of consistency in the past and is expected to continue to do so in the future. This consistency is characterized by a stream of data falling within control limits based on plus or minus 3 standard deviations (3 sigma) of the centerline. (Hachicha and Ghorbel, 2012)

The main aim behind the idea of control charts is the need for perfection and elimination of non-conforming products. Control chart helps to differentiate between the inherent variation in a process and variation due to assignable causes. The inherent variation in a process is background noise due to several small unavoidable causes. Assignable causes are considerably larger fluctuations when compared to the background noise. Variation from an assignable cause can only be removed from the process through human intervention (Juran and Godfrey, 1998).

Control charts are classified by the type of quality characteristic they are supposed to monitor. Control charts can be broadly classified as control charts for variables and control charts for attributes.

One of the first control charts to receive attention is the $\overline{X}$ chart, devised by Walter Shewhart. The $\overline{X}$ chart provides an illustrative example for general control chart theory. The $\overline{X}$ control chart consists of a centre line (CL or μ_0), an upper control limit (UCL) and a lower control limit (LCL).

In $\overline{\overline{X}}$ control chart, the sample mean is compared with the upper and lower control limits of the control chart to decide whether the process is in-control or out-of-control. If a point falls within the upper and lower control limits, the process is referred to as "in control" whereas if it falls outside the control limits, the process is referred to as "out-of control". There are two possible errors: a process can be deemed in-control when in fact the process is out-of-control (Type II error), and vice versa (Type I error). When the process is judged to be out-of-control, there is an attempt to identify the special cause of variation which is called an assignable cause search. (Duncan, 1956)

Generally there are economic design of $\overline{X}$ control chart and economic statistical design of $\overline{X}$ control chart. In economic design of $\overline{X}$ control chart, the objective is to reduce the total cost of maintaining the control chart as minimum as possible. It is used to determine the values of various design parameters i.e. sample size (n), sampling interval (h), and control limit coefficient (L) that minimizes total expected cost. The statistical errors associated with control chart are Type-I error and Type-II error. These two errors are cannot be completely eliminated since 100% inspection is not carried out. In economic statistical design, the total cost of maintaining the control chart need to be minimized and at the same time Type-I and Type-II errors are not allowed to exceed their permissible level.

The remainder of this paper is organized as follows: Section II presents the problem description. The proposed genetic algorithm is explained in section III. Result and discussion are presented in section IV and section V gives the concluding remarks.

II. Problem Description

The customer requirements and expectations are becoming increasingly high in terms of quality and cost in the present industrial environment. Accordingly the selection of control chart design parameters like n, h and L becomes a challenging job. The economic statistical design of $\overline{X}$ control chart is considered in this paper to determine the parameters of $\overline{X}$ control chart.

A. Mathematical model

The mathematical model for economic statistical design of the $\overline{X}$ control chart is adopted from Rahim and Banerjee cost's model proposed in 1993. In this model, the failure mechanism belongs to the gamma (λ, 2) distribution, and the sample mean $\overline{X}$ is normally distributed.

1) The cost model of uniform sampling interval (Rahim and Banerjee, 1993)

The objective function of the cost model is expressed mathematically as:

$$\text{Min } F(n, h, L) = \frac{E(C)}{E(T)} \quad (1)$$

$$\text{Subject to } \alpha \leq \alpha_{U},\ 1-\beta \geq P_{L}, \text{ and} \quad (2)$$

$$n \geq 1 \text{ and integer},\ h \geq 0,\ L \geq 0 \quad (3)$$

Where E(T) and E(C) represent the expected cycle length and the total expected cost per cycle, respectively. The objective function that the type I error probability (α) and power (1-β) as subjected to the predetermined statistical constraints, including maximum value of type I error (α_U) and minimum value of power (P_L), is minimized by determining the sample size (n), sampling interval (h), and the control limits (L). This can be expressed mathematically as:

$$E(T) = h + (\alpha Z_0 + h)\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\left(1 + \frac{\lambda h}{1-e^{-\lambda h}}\right) + \frac{h\beta}{1-\beta} + Z_1 \quad (4)$$

$$E(C) = (a + bn + \alpha Y + D_1 h)\frac{e^{-\lambda h}}{1-e^{-\lambda h}}\left(1 + \frac{\lambda h}{1-e^{-\lambda h}}\right) + \frac{a+bn}{1-\beta} + \frac{2D0}{\lambda} + D_1\left(\frac{\beta}{1-\beta} - \frac{\lambda}{2}\right) + W \quad (5)$$

$$\text{Where } \alpha = 2\emptyset(-L),\ \beta = 1-\left[\emptyset\left(\delta\sqrt{n} - L\right) + \emptyset\left(-\delta\sqrt{n} - L\right)\right] \quad (6)$$

2) The cost model of non-uniform sampling interval(Rahim and Banerjee, 1993)

The objective function of the cost model is expressed mathematically as:

$$\text{Min } F(n, h_1, h_2, L) = \frac{E(C)}{E(T)} \tag{7}$$

$$\text{Subject to } \alpha \leq \alpha_{U},\ 1\text{-}\beta \geq P_{L}, \text{ and} \tag{8}$$

$$n \geq 1 \text{ and integer},\ h_1 \geq 0,\ h_2 \geq 0,\ L \geq 0 \tag{9}$$

Where E(T) and E(C) represent the expected cycle length and the total expected cost per cycle, respectively. The objective function that the type I error probability (α) and power (1-β) as subjected to the predetermined statistical constraints, including maximum value of type I error (α_U) and minimum value of power (P_L), is minimized by determining the sample size (n), sampling interval (h_1, h_2, representing the intervals of drawing a sample initially and taking a sample after the first sample over the cycle length, respectively), and the control limits (L). This can be expressed mathematically as:

$$E(T) = h_1 + (\alpha Z_0 + h_2)\frac{e^{-\lambda h_1}}{1-e^{-\lambda h_2}}\left(1+\lambda h_1+\frac{\lambda h_2 e^{-\lambda h_2}}{1-e^{-\lambda h_2}}\right)+\frac{h_2\beta}{1-\beta}+Z_1 \tag{10}$$

$$E(C) = (a + bn + \alpha Y + D_1 h_2)\frac{e^{-\lambda h_1}}{1-e^{-\lambda h_2}}\left(1+\lambda h_1+\frac{\lambda h_2 e^{-\lambda h_2}}{1-e^{-\lambda h_2}}\right)+\frac{a+bn}{1-\beta}+\frac{2D_0}{\lambda}+D_1\left(\frac{h_2\beta}{1-\beta}-\frac{2}{\lambda}\right)+W \tag{11}$$

$$\text{Where } \alpha = 2\emptyset(-L),\ \beta = 1-\left[\emptyset\left(\delta\sqrt{n}-L\right)+\emptyset\left(-\delta\sqrt{n}-L\right)\right] \tag{12}$$

The parameters of the model are listed below.

Time parameters:

Z_0 = the expected search time associated with a false alarm

Z_1 = the expected search time and repair time if a failure is detected

Cost parameters:

D_0 = the expected cost per hour caused by the production of a nonconforming item when the process is in control

D_1 = the expected cost per hour caused by the production of a nonconforming item when the process is out of control

W = the expected cost of locating an assignable cause and repairing the process, including the cost of down time

Y = the expected cost of false alarms, including the costs of searching and down time if production ceases during the search

a = the fixed cost per sample

b = the cost per unit sample

III. Methodology of Proposed Genetic Algorithm

Genetic algorithms (GA) are the heuristic search and optimization techniques that mimic the process of natural evolution. Simplicity of operation and power of effect are two of the main attractions of the GA approach (Goldberg, 1989). GA can be applied to a wide range of problems (e.g. location, partitioning, and scheduling problems) and GA makes no assumptions about the functions to be optimized.

All that GA requires is a performance measure, some form of population representation, and operators that generate new population members. This general approach can be applied to many combinatorial optimization problems. Hence GA is adapted for the economic statistical design of $\bar{X}$ control chart in this study. Adaptation is made with respect to chromosome representation, population initialization, crossover operation, and mutation operation in the proposed GA.

1) Mechanism of the proposed genetic algorithm (ESDCC-GA)

The proposed genetic algorithm is shown in Figure 1

a. Initialization

Initialize population size (N), crossover rate (CR) and mutation rate (MR) for the proposed genetic algorithm.

b. Initial population generation and fitness evaluation

Initial population of size N is randomly generated under a constrained condition for uniform sampling interval and non-uniform sampling interval.

c. Chromosome structure and representation

Each individual in a population is called chromosome (C). In the current study the problem is to optimize the parameters of control chart for uniform (n, h, L) and non-uniform (n, h_1, h_2, L) sampling interval scheme to reduce expected cost per hour (ECT). So each chromosome is coded with parameters of control chart. Chromosome representation in this case is phenotype, that is actual values of parameters are used to code all the genes in a chromosome. The structure of chromosome for uniform sampling interval is shown in Table 1 and non – uniform sampling interval is shown in Table 2. The chromosome length (*l*) is set as equal to number of parameters.

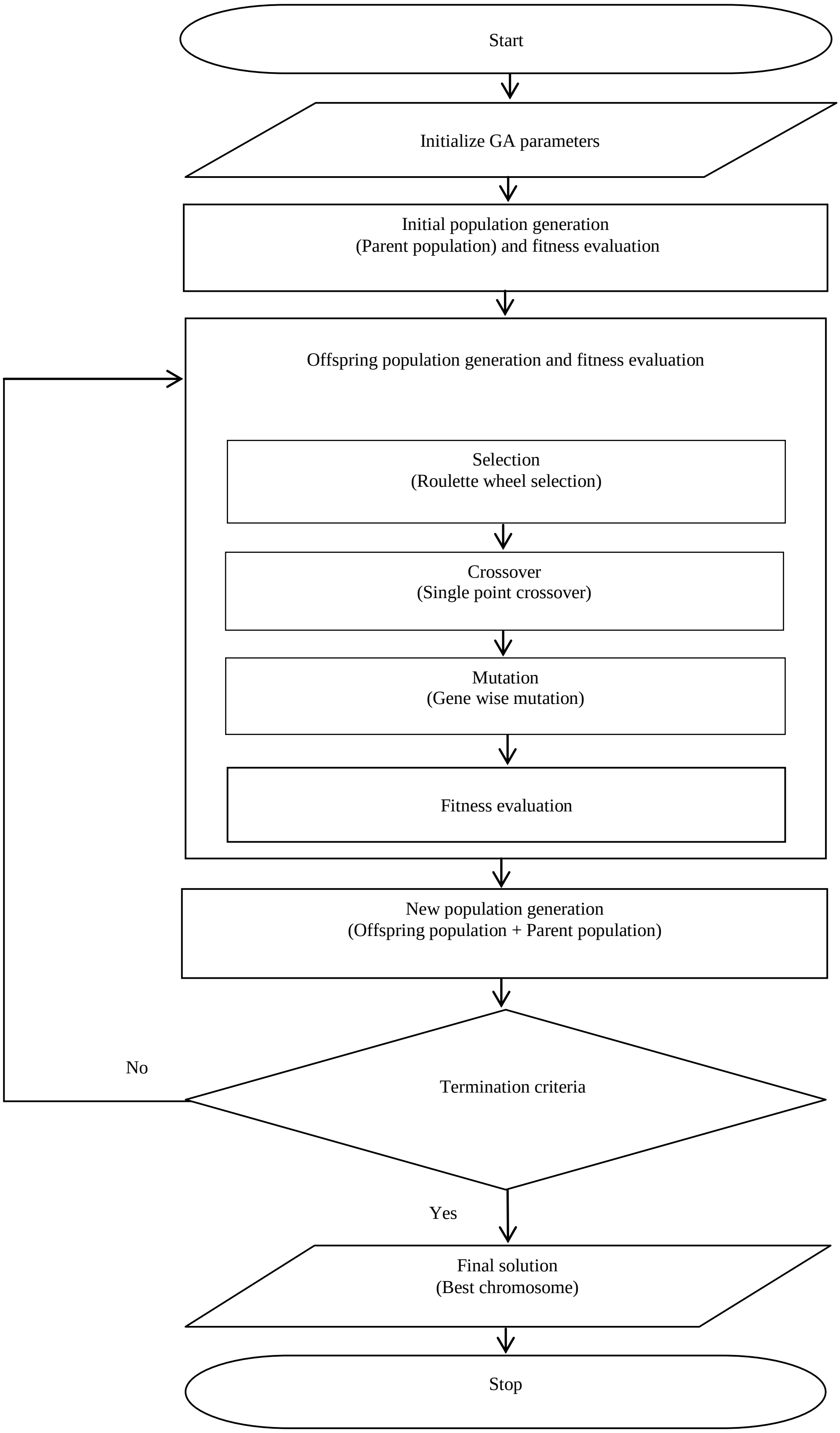

Figure 1 Proposed genetic algorithm

In the case of sample size, the generated gene value should be discrete, and for sampling interval and control limit, gene value is continuous for each chromosomes.
Gene value of each chromosome is generated using the following equation (Daniel and Rajendran, 2005).
Gene= rand ()* (upper_limit - lower_limit) + lower_limit;

Table 1 Chromosome structure for uniform sampling interval scheme

Chromosome, C		
g_1	g_2	g_3
n	h	L

Table 2 Chromosome structure for non-uniform sampling interval scheme

Chromosome, C			
g_1	g_2	g_3	g_4
n	h_1	h_2	L

d. Fitness evaluation
Every chromosome in the initial population is evaluated with respect to its fitness function, F = 1/ECT.
e. Offspring population generation and fitness evaluation
Offspring populations are generated by applying the following GA operators.
1 Selection - roulette wheel procedure
2 Crossover - single point crossover (based on crossover rate)
3 Mutation - gene wise mutation (based on mutation rate)
The fitness value of each chromosome in the offspring population is also calculated.
f. New population generation
The chromosomes in the offspring and parent populations are combined to generate a new population . The best N chromosomes, among the 2N chromosomes are chosen as the surviving chromosomes for the next generation (Parental chromosomes for the next generation).
g. Termination criteria and final solution
The total number of generation is taken as the termination criteria and GA gives the global best solution after termination criteria is satisfied.

IV. Results and Discussion

A. Economic statistical design of $\bar{X}$ control chart under uniform sampling interval

In the computational experiment of uniform sampling interval, the population size was set to 80. The crossover probability and mutation probability were set to 0.7 and 0.2, respectively. Parameters are selected based on the pilot study. Generation number is selected based on the convergence analysis of each test problem. For uniform sampling interval, the initial population is randomly generated under the following constrained condition.

$1 \leqslant n \leqslant 3000$ (Sample size)
$0.1 \leqslant h \leqslant 100$ (Sampling interval)
$0.1 \leqslant L \leqslant 6$ (Control limit)

The values of time, cost, Gamma and shift parameters of the example test problem are as follows: Z_0 = 0.25 h; Z_1 = 1.00 h; D_0 = \$50.00; D_1 = \$950.0; W = \$1100.00; Y = \$500.00; a = \$20.00; b = \$4.22; δ = 0.50; λ = 0.05; alphaUB = 0.05; and pLB = 0.9. After the experimental study, convergence point of the test problem is identified as 77901th generation and the generation number was set to 80000. The result obtained from ESDCC-GA is compared with the result obtained for same number of solutions (3921017) from PSO (Chih et. al) and it is shown in Table 3. The result shows that EDDCC-GA is better than PSO in terms of minimum ECT and is faster than PSO in terms of elapsed time for uniform sampling interval.

Table 3 Result of uniform sampling interval scheme by ESDCC-GA

Algorithm	n	h	L	α	1-β	ECT	Time (m)
ESDCC-GA	43	4.3879	1.9599	0.05	0.9063	178.0005	29.28
PSO	43	4.3364	1.9602	0.0499	0.9063	178.0085	48.82

B. Economic statistical design of $\bar{X}$ control chart under non-uniform sampling interval

In the computational experiment of non-uniform sampling interval, the population size was set to 80. The crossover probability and mutation probability were set to 0.7 and 0.2, respectively. Parameters are selected after conducting the pilot study. Generation number of each test problem under non-uniform sampling interval is selected based on convergence analysis. For non - uniform sampling interval, the initial population is randomly generated under the following constrained condition.

$1 \leqslant n \leqslant 3000$ (Sample size)
$0.1 \leqslant h1 \leqslant 100$, $0.1 \leqslant h2 \leqslant 40$ (Sampling interval)
$0.1 \leqslant L \leqslant 6$ (Control limit)

The values of time, cost, Gamma (λ, 2), and shift parameters of the example are as follows: $Z_0 = 0.25$ h; $Z_1 = 1.00$ h; D_0 = \$50.00; D_1 = \$950.0; W = \$1100.00; Y = \$500.00; a = \$20.00; b = \$4.22; $\delta = 0.50$; $\lambda = 0.05$; alphaUB = 0.05; and pLB = 0.9. In the computational experiments of non-uniform sampling interval, convergence point of the test problem is identified as 79946th generation and the generation number was set to 90000. The result obtained from ESDCC-GA is compared with the result obtained for same number of solutions (5062252) from PSO (Chih et. al) and it is shown in Table 4. The result shows that EDDCC-GA is better than PSO in terms of minimum ECT and is faster than PSO in terms of elapsed time for non uniform sampling interval.

Table 4 Result of non-uniform sampling interval scheme

Algorithm	n	h_2	h_2	L	α	$1-\beta$	ECT	Time (m)
ESDCC-GA	43	10.53	3.9081	1.9599	0.05	0.9063	173.803765	36.04
PSO	43	10.60	3.8589	1.9647	0.0494	0.9055	173.8344	72.38

C. Test problems

To demonstrate the efficacy of the proposed GA, 20 test problems from literature (Chih et. al) were solved using ESDCC – GA. The test problems considered in this study are shown in Table 5.

D. Result of test problems

The results of test problems by ESDCC-GA and PSO (Chih et. al) under uniform and non-uniform sampling interval are shown in Table 6 and Table 7 respectively. The result shows that ESDCC-GA takes lesser time than PSO (Chih et. al) for inspecting similar number of solutions in uniform and non-uniform sampling interval. The result indicates that ESDCC-GA is superior to PSO (Chih et. al) in terms of convergence speed for economic statistical design of $\overline{X}$ control chart.

The proposed GA is better than PSO in terms of minimum ECT and is faster than PSO in terms of elapsed time. Comparing the uniform and non-uniform sampling interval scheme, the expected cost per hour (ECT) is minimum in non-uniform sampling interval scheme.

V. CONCLUSION

This present study aimed to develop a genetic algorithm (ESDCC-GA) for economic statistical design of $\overline{X}$ control charts under uniform and non-uniform sampling interval. The proposed algorithm is designed to solve the constrained problem, which involves the simultaneous use of continuous and discrete decision variables. To verify the performance of the proposed GA, the numerical example of Rahim and Banerjee (1993) with a Gamma failure mechanism is illustrated in this paper.

The various test problems in the literature is also used to evaluate the proposed GA. The computational results demonstrated that proposed GA and PSO (Chih et. al) have the similar performance in terms of final solution quality for the economic statistical design of $\overline{X}$ control charts. However, the proposed GA (EDDCC-GA) is significantly faster and is better than PSO (Chih et. al) in terms of elapsed time. When inspecting similar number of solutions, EDDCC-GA takes lesser time than PSO (Chih et. al). A higher cost was saved in the non-uniform sampling scheme than the uniform sampling scheme regardless of which method was adopted. Meanwhile, no significant differences were observed between control chart parameters from EDDCC-GA and PSO (Chih et. al).

Table 5 Test problems

Problem No.	Z_0	Z_1	D_0	D_1	W	Y	a	b	δ	α_{UB}	p_{LB}	λ
1	0.025	0.1	25	475	550	250	10	2.11	0.25	0.01	0.85	0.025
2	0.025	0.1	25	475	1100	500	20	4.22	0.5	0.05	0.9	0.05
3	0.025	0.1	25	475	2200	1000	40	8.44	1	0.1	0.95	0.1
4	0.025	1	50	950	550	250	10	4.22	0.5	0.05	0.95	0.1
5	0.025	1	50	950	1100	500	20	8.44	1	0.1	0.85	0.025
6	0.025	1	50	950	2200	1000	40	2.11	0.25	0.01	0.9	0.05
7	0.025	10	100	1900	550	250	10	8.44	1	0.1	0.9	0.05
8	0.025	10	100	1900	1100	500	20	2.11	0.25	0.01	0.95	0.1
9	0.025	10	100	1900	2200	1000	40	4.22	0.5	0.05	0.85	0.025
10	0.25	0.1	50	1900	550	500	40	2.11	0.5	0.1	0.85	0.05
11	0.25	0.1	50	1900	1100	1000	10	4.22	1	0.01	0.9	0.1
12	0.25	0.1	50	1900	2200	250	20	8.44	0.25	0.05	0.95	0.025
13	0.25	1	100	475	550	500	40	4.22	1	0.01	0.95	0.025
14	0.25	1	100	475	1100	1000	10	8.44	0.25	0.05	0.85	0.05
15	0.25	1	100	475	2200	250	20	2.11	0.5	0.1	0.9	0.1
16	0.25	10	25	950	550	500	40	8.44	0.25	0.05	0.9	0.1
17	0.25	10	25	950	1100	1000	10	2.11	0.5	0.1	0.95	0.025
18	0.25	10	25	950	2200	250	20	4.22	1	0.01	0.85	0.05
19	0.5	0.1	100	950	550	1000	20	2.11	1	0.05	0.85	0.1
20	0.5	0.1	100	950	1100	250	40	4.22	0.25	0.1	0.9	0.025

Table 6 Result of test problem under uniform sampling interval scheme

Problem No.		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
GA	Convergence Point	76666	65126	73023	64956	82808	81671	82117	76667	78352	67187
	Generation Number	80000	70000	80000	70000	90000	90000	90000	80000	80000	70000
	Solutions inspected	3925996	3393136	3816918	3407686	4289034	4542012	4363175	3874998	3836693	3428670
	n	209	43	14	52	10	239	9	286	39	41
	h	12.1287	6.53722	5.16334	3.44407	4.39791	7.25411	2.01607	3.81182	4.31569	2.40646
	L	2.5758	1.95996	2.0968	1.95996	1.98869	2.57583	1.71844	2.57583	1.97777	1.92076
	α	0.01	0.05	0.03601	0.05	0.04674	0.01	0.08572	0.01	0.04795	0.05476
	1-β	0.85045	0.90637	0.95	0.95008	0.85972	0.90131	0.9	0.95074	0.87384	0.89987
	ECT	110.293	124.136	212.466	213.171	120.963	259.434	176.765	334.421	218.452	190.625
	Time (m)	40.5602	35.4594	39.4241	39.9938	41.5442	41.8979	41.8379	39.3925	39.8306	35.1188
PSO	Solutions inspected	3925996	3393136	3816918	3407686	4289034	4542012	4363175	384998	3836693	3428670
	n	210	43	14	52	10	240	9	286	39	41
	h	12.2792	6.65081	5.13963	3.3402	4.40073	7.2783	2.02334	3.75501	4.3125	2.40402
	L	2.57834	1.96014	2.09583	1.96033	1.98708	2.5786	1.7164	2.57619	1.97471	1.91979
	α	0.00993	0.04998	0.0361	0.04996	0.04691	0.00991	0.08609	0.00999	0.0483	0.05488
	1-β	0.85188	0.90635	0.9501	0.95004	0.88004	0.90229	0.90036	0.9507	0.87447	0.90004
	ECT	110.359	124.146	212.472	213.231	120.963	259.427	176.769	334.452	218.452	190.625
	Time (m)	52.5044	43.0607	61.7003	47.8221	59.1871	65.2915	62.0128	66.5141	59.9515	42.4805

Table 6 Result of test problem under uniform sampling interval scheme (Continued)

Problem No.		11	12	13	14	15	16	17	18	19	20
GA	Convergence Point	66789	73686	84124	58117	77680	75551	86264	74332	65123	81671
	Generation Number	70000	80000	90000	60000	80000	80000	90000	80000	70000	90000
	Solutions inspected	3366393	3809401	4371420	2896750	3989404	3924229	4289645	3742404	3243902	4561267
	n	15	208	18	144	35	169	59	14	17	138
	h	1.27861	12.9625	7.22725	19.1451	4.10507	8.4715	5.22212	2.47284	1.6928	10.8103
	L	2.57583	1.95997	2.57583	1.95997	1.64485	1.9623	2.1957	2.5758	2.6864	1.64485
	α	0.01	0.05	0.01	0.05	0.1	0.04	0.02811	0.01	0.00722	0.1
	1-β	0.90271	0.95008	0.95222	0.85084	0.90544	0.90148	0.95	0.8781	0.92459	0.9018
	ECT	235.193	364.356	139.278	272.858	254.426	294.623	90.2557	117.582	201.632	235.68
	Time (m)	38.6242	39.7378	46.0379	22.9571	37.8936	40.46	47.2025	33.4396	37.1808	43.1536
PSO	Solutions inspected	3366393	3809401	4371420	2896750	3989404	3924229	4289645	3742404	3243902	4561267
	n	15	209	18	144	35	169	59	14	17	138
	h	1.28831	13.2154	7.25323	18.4653	4.0838	8.4715	5.2575	2.47846	1.70304	10.7823
	L	2.57679	1.96	2.57872	1.96298	1.64517	1.96237	2.19245	2.57799	2.6843	1.64527
	α	0.00997	0.04976	0.00992	0.04965	0.09993	0.04972	0.0283	0.00994	0.00727	0.09991
	1-β	0.90255	0.95075	0.95194	0.85014	0.90538	0.90106	0.95034	0.87777	0.92489	0.90174
	ECT	235.199	364.849	139.283	273.004	254.427	294.735	90.2746	117.602	201.634	235.688
	Time (m)	46.4415	60.0953	61.4228	26.843	62.6017	59.2417	63.546	52.425	45.3988	63.8143

Table 7 Result of test problem under non-uniform sampling interval scheme

Problem No.		1	2	3	4	5	6	7	8	9	10
GA	Convergence Point	56794	71972	71357	67896	79952	73958	73958	72000	79951	62432
	Generation Number	60000	80000	80000	70000	90000	90000	80000	80000	80000	70000
	Solutions inspected	3541384	4495917	4537386	3954281	4970392	5054985	4395712	4549270	4426780	3975804
	n	209	43	14	52	10	239	9	286	38	40
	h_1	27.30075	13.62814	8.875577	6.789171	13.97242	14.65416	6.50242	7.25993	13.86627	7.238837
	h_2	10.40099	5.63661	4.29653	2.984342	4.048314	6.185903	1.873377	3.277038	3.925159	2.189877
	L	2.57583	1.959965	2.096802	1.959964	1.996788	2.57583	1.71844	2.575832	1.974716	1.917124
	α	0.01	0.05	0.036011	0.05	0.045848	0.01	0.085716	0.01	0.0483	0.055222
	1-β	0.850453	0.906374	0.95	0.950076	0.878089	0.901314	0.9	0.950738	0.865959	0.893462
	ECT	106.5418	120.9353	209.0958	207.3724	118.9528	252.3095	173.9508	324.4844	214.7907	186.2246
	Time (m)	25.6872	30.9271	31.0345	26.1945	35.7714	35.4539	37.5203	30.1581	32.5003	29.0492
PSO	Solutions inspected	3541384	4495917	4537386	3954281	4970392	5054985	4395712	4549270	4426780	3975804
	n	209	43	14	53	10	241	9	286	39	40
	h_1	26.18701	13.66926	8.993446	6.882441	13.9249	14.08294	6.376289	7.382344	13.93939	7.347877
	h_2	10.96082	5.724924	4.206441	3.040485	4.038741	6.161699	1.860838	3.139819	3.990992	2.174579
	L	2.577271	1.960872	2.096268	1.965662	2.00214	2.588259	1.717832	2.576188	1.986138	1.922985
	α	0.009958	0.049894	0.036058	0.049338	0.04527	0.009646	0.085827	0.00999	0.047018	186.2294
	1-β	0.850117	0.906223	0.950055	0.952973	0.877004	0.901957	0.900107	0.950702	0.872097	0.054482
	ECT	106.6491	120.9449	209.1115	208.1076	118.9533	252.7861	173.9574	324.6389	214.7918	0.892381
	Time (m)	47.33	54.61	56.6563	46.4123	70.946	76.7032	54.0317	82.4915	56.6679	46.8512

Table 7 Result of test problem under non-uniform sampling interval scheme (Continued)

Problem No.		**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
GA	Convergence Point	67507	71986	71990	72456	79648	74298	82457	70000	80000	80000
	Generation Number	70000	80000	90000	80000	90000	80000	90000	67234	75345	76479
	Solutions inspected	3962218	4605403	5074088	4523146	5024789	4534782	5121748	392476	4487415	4612481
	n	15	208	18	144	35	169	67	15	17	138
	h_1	3.755649	26.09059	17.85138	29.0862	7.9063	12.998	14.631	7.5664	4.4	24.194
	h_2	1.175287	11.28959	6.559405	13.8705	3.4683	6.5424	4.8716	2.386	1.5227	9.4628
	L	2.57583	1.959966	2.575829	1.96	1.644	1.96	2.4478	2.5758	2.6989	1.644
	α	0.01	0.05	0.01	0.05	0.01	0.05	0.014	0.01	0.007	0.1
	1-β	0.902711	0.950075	0.952224	0.8507	0.9053	0.9014	0.95	0.9027	0.9228	0.9017
	ECT	230.5957	352.6095	138.0564	265.8333	251.9064	282.5949	89.1105	115.7021	198.8551	230.4388
	Time (m)	30.0996	32.195	35.752	33.45	36.12	32.78	36.478	28.367	33.147	34.128
PSO	Solutions inspected	3962218	4605403	5074088	4523146	5024789	4534782	5121748	392476	4487415	4612481
	n	15	209	18	145	35	169	62	15	17	139
	h_1	0.1	26.68258	17.888	29.1032	7.8979	13.0407	14.8415	7.5292	4.3771	23.987
	h_2	1.283045	11.26566	6.5765	13.918	3.4691	6.5984	4.772	2.2545	1.558	9.3645
	L	2.577229	1.967786	2.575	1.96	1.63	1.96	2.2554	2.568	2.6651	1.6449
	α	0.00996	0.049093	0.01	0.05	0.01	0.05	0.024	0.01	0.0071	0.09
	1-β	0.90247	0.950161	0.9522	0.8507	0.90541	0.90142	0.9503	0.8782	0.9213	0.9507
	ECT	238.8497	353.2209	139.1245	267.0124	252.3654	282.9012	89.8742	115.7841	199.1245	230.784
	Time (m)	53.3333	63.3	66.412	60.124	69.321	58.651	71.23	52.198	59.34	61.147

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