A Transportation Problem Analysed by a New Ranking Method

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Abstract—.**This paper describes a ranking method of hexagonal fuzzy numbers using the area of trapezium. This ranking is used in a transportation problem in which the values of transportation costs are represented as hexagonal fuzzy numbers and then the fuzzy transportation problem(FTP) is converted into a crisp valued Transportation Problem using a new ranking. A numerical example is applied to obtain an initial Basic Feasible solution of transportations in minimizing transportation time by North-West Corner method.**

Keywords- Ranking; Hexagonal fuzzy numbers; Fuzzy Transportation Problem.

I. INTRODUCTION

In a fuzzy environment ranking fuzzy numbers is an important decision making procedure. Most of the real world problems that exist are fuzzy in nature, than probabilistic or deterministic. Fuzzy sets was first given by Zadeh [15] in 1965. Ranking fuzzy numbers was first proposed by Jain in the year 1976 for decision making in fuzzy situations by representing in fuzzy set. Jain [6] proposed a method using the concept of maximizing set to order the fuzzy numbers and the decision maker considers only the right side membership function. Yager [14] proposed four indices to order fuzzy quantities in [0, 1]. Liou and Wang [9] presented ranking fuzzy numbers with integral value. Centroid based distance method to rank fuzzy numbers was given by Cheng [4] in 1998. Wang and Kerre [13] classified the existing ranking procedures into three classes. The first class consists of ranking procedures based on fuzzy mean and spread and second class consists of ranking procedures based on fuzzy scoring whereas, the third class consists of methods based on preference relations and concluded that the ordering procedures associated with first class are relatively reasonable for the ordering of fuzzy numbers specially. The ranking procedure presented by Adamo [2] satisfies all the reasonable properties for the ordering of fuzzy quantities. The Ranking of fuzzy numbers by area between the centroid point and the original point was given by Chu and Tsao [5] in 2002.The fuzzy risk analysis based on ranking of generalized trapezoidal fuzzy numbers was given by Chen and Chen [3]. Abbasbandy and Hajjari [1] proposed a new approach for ranking trapezoidal fuzzy numbers. Stephen Dinagar and Kamalanathan [12] defined a ranking based on area method. Since then several methods have been proposed by various researchers which includes rank, mode, divergence and spread [8], Rajarajeshwari and Sahaya Sudha [10] defined Ranking of Hexagonal fuzzy numbers using centroid.

II. PRILIMINARIES

2.1 Definition [7]

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function A: X →[0, 1], where A(x) is interpreted as the degree of membership of element x in fuzzy A for each x ∈ X.

2.2 Definition: [7]

The characteristic function $\mu_{\bar{A}}(x)$ of a crisp set A ⊆ X assigns a value of either 1 or 0 to each individual in the universal set X .This function can be generalized to a function $\mu_{\bar{A}}$ such that the value assigned to the element of the universal set X fall within a specified rage (i.e.) $\mu_{\bar{A}}$: X →[0, 1] . The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$ defind by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set

2.3 Definition: [7]

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be a fuzzy number of its membership function has the following characteristics

(i) $\mu_{\tilde{A}}(x)$ is continuous mapping from R to the closed interval [0,1]
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a_1] \cup [a_4, \infty]$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$
(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a_3, a_4]$ where $a_1 \leq a_2 \leq a_3 \leq a_4$

2.4 Definition: [7]

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be a generalized fuzzy number of its membership function has the following characteristics

(i) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a_1] \cup [a_4, \infty]$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$
(iv) $\mu_{\tilde{A}}(x) = w$ for all $x \in [a_2, a_3]$ where $0 < w \leq 1$

2.5 Definition: [7]

A fuzzy number A is a trapezoidal fuzzy number denoted by (a_1, a_2, a_3, a_4) and its membership function is given below, where $a_1 \leq a_2 \leq a_3 \leq a_4$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if\ a_1 \leq x \leq a_2 \\ 1 & if\ a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & if\ a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

2.6 Definition: [7]

A generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w(\frac{x-a_1}{a_2-a_1}) & if\ a_1 \leq x \leq a_2 \\ w & if\ a_2 \leq x \leq a_3 \\ w(\frac{a_4-x}{a_4-a_3}) & if\ a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

Hexagonal Fuzzy Numbers

2.7 Definition: [11]

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H\,(a_1, a_2, a_3, a_4 a_5, a_6,)$ where $a_1, a_2, a_3, a_4 a_5, a_6$ are real numbers and its membership function $\mu_{\tilde{A}_H}$ (x) is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \le x \le a_3 \\ 1, & \text{for } a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & \text{for } a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases}$$

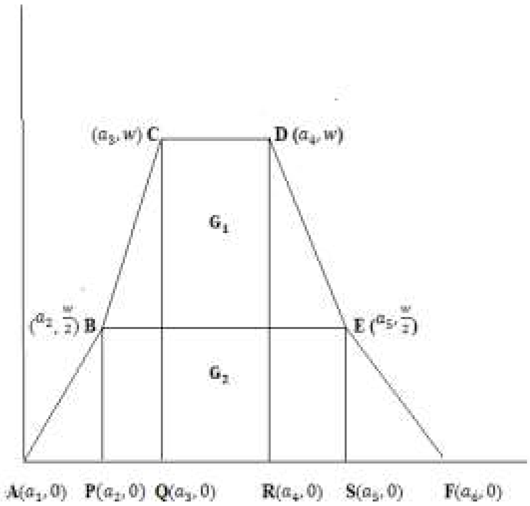
2.8 Definition: [11]

A generalized fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4 a_5, a_6, w)$ where $(a_1, a_2, a_3, a_4 a_5, a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & \text{for } x \prec a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}w\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \le x \le a_3 \\ w, & \text{for } a_3 \le x \le a_4 \\ 1-\frac{w}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & \text{for } a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases}$$

III. Proposed Approach

In this paper we have used the area of trapezium as a new ranking procedure for generalized trapezoidal fuzzy numbers. Divide the hexagon into two trapezium G_1 and G_2 and each trapezium has two bases parallel to each other and two legs joining these bases at the points of G_1 are (a_3, w), (a_4, w), $(a_2, w/2)$, $(a_5, w/2)$ and G_2 are $(a_1, 0)$, $(a_6, 0)$, $(a_2, w/2)$, $(a_5, w/2)$.



Area of trapezium $G_1 = \frac{w}{2}(b_1 + b_2)$

b_1 = distance between B $(a_2, {}^{w}/_{2})$ and E $(a_5, {}^{w}/_{2})$

$$b_1 = \sqrt{(a_5 - a_2)^2 + ({}^{w}/_{2} - {}^{w}/_{2})^2} = (a_5 - a_2)$$

b_2 = distance between C (a_3, w) and D (a_4, w)

$$b_2 = \sqrt{(a_4 - a_3)^2 + (w - w)^2} = (a_4 - a_3)$$

$$G_1 = \frac{w}{2}[(a_5 - a_2) + (a_4 - a_3)]$$

Area of trapezium $G_2 = \frac{w}{2}(b_1 + b_3)$

b_1 = distance between B $(a_2, {}^{w}/_{2})$ and E $(a_5, {}^{w}/_{2})$

$$b_1 = \sqrt{(a_5 - a_2)^2 + ({}^{w}/_{2} - {}^{w}/_{2})^2} = (a_5 - a_2)$$

b_3 = distance between A $(a_1, 0)$ and F $(a_6, 0)$

$$b_3 = \sqrt{(a_6 - a_1)^2 + (0 - 0)^2} = (a_6 - a_1)$$

$$G_2 = \frac{w}{2}[(a_5 - a_2) + (a_6 - a_1)]$$

Hence the ranking function is given by

$R(\tilde{A}) = G_1 + G_2$

$$R(\tilde{A}) = \frac{w}{2}[(a_5 - a_2) + (a_4 - a_3)] + \frac{w}{2}[(a_5 - a_2) + (a_6 - a_1)]$$

$$R(\tilde{A}) = \frac{w}{2}[(a_6 - a_1) + 2(a_5 - a_2) + (a_4 - a_3)] \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ } \mathbf{(3.1)}$$

IV. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM

The mathematical models of fuzzy transportation problem is to minimize the total transportation cost from m Machine to n Foundries is as follows

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}$$

$$Subject\quad to\ \sum_{j=1}^{m}\tilde{x}_{ij} = a_i\ ,\qquad i = 1,2,3,.........,m$$

$$\sum_{i=1}^{n}\tilde{x}_{ij} = b_j\ ,\qquad j = 1,2,3,.........,n$$

$$\sum_{i=1}^{m}a_i = \sum_{j=1}^{n}\ b_j\ ,$$

$$and\ \ \tilde{x}_{ij} \geq 0\quad for\ all\ i\ and\ j.$$

Where $\tilde{c}_{ij}$ the fuzzy unit transportation cost from i[th] Machines to the j[th] Foundries.
The fuzzy transportation table is:

Table 4.1: Fuzzy Transportation Table

MACHINES	FOUNDRIES					
		1	2	…	N	Supply
	1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\cdots$	$\tilde{c}_{1n}$	a_1
	2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\cdots$	$\tilde{c}_{2n}$	a_2
	.	.	.	…	.	.
	M	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	$\cdots$	$\tilde{c}_{mn}$	a_m
	Demand	b_1	b_2	$\cdots$	b_n	

V. THE ALGORITHM FOR THE PROPOSED WORK IS AS FOLLOWS

Step 1:
Construct the fuzzy transportation table for the given problem and then convert into a balanced, if it is not.

Step 2:
Using the above ranking method **(3.1)** the fuzzy transportation problem is converted into a crisp problem.

Step 3:
Apply North West Corner (NWC) method to obtain an initial basic feasible solution.

Step 4:

1) Select the north-west (upper left hand) corner cell of the transportation table and allocate as much as possible so that either the capacity of the first row or first column is satisfied i.e., x_{11} = min (a_1 , b_1).
2) If $b_1 > a_1$, we move down vertically to the second row and make the second allocation of magnitude x_{21} = min $(a_2 , b_1 - x_{11})$ in the cell (2, 1).
   If $b_1 < a_1$, we move right horizontally to the second column and make the second allocation of magnitude x_{12} = min $(a_1 - x_{11} , b_2)$ in the cell (1, 2).
   If $b_1 = a_1$, there is a tie for the second allocation. We can make the second allocation of magnitudes
   x_{12} = min $(a_1 - a_1 , b_2) = 0$ in the cell (1, 2).
   Or x_{21} = min $(a_2 , b_1 - b_1) = 0$ in the cell (2, 1).

Step 5:

Repeat the above steps moving down towards the lower/ right corner of the transportation table until the optimal solution is obtained.

VI. Numerical Example

A company manufactures certain type of products in three different Machines. The company has to execute the transportation of the three Machines to four different Foundries. The information about the cost of transportation is imprecise and here hexagonal fuzzy numbers are used to represent the cost. The fuzzy transportation problem is given in table 6.1

Step 1:

Construct the fuzzy transportation table for the given fuzzy transportation problem and then convert it into a balanced one, if it is not.

TABLE 6.1 FUZZY TRANSPORTATION TABLE

Foundries / Machines	F_1	F_2	F_3	F_4	Supply
M_1	(3,7,11,15, 19,24;0.5)	(13,18,23,28, 33,40;0.7)	(6,13,20,28, 36,45;0.4)	(15,20,25,31, 38,45;0.8)	16
M_2	(16,19,24,29 34,39;0.2)	(3,5,7,9, 10,12;0.5)	(5,7,10,13, 17,21;0.6)	(20,23,26,30, 35,40;0.4)	36
M_3	(11,14,17,21, 25,30;0.7)	(7,9,11,14, 18,22;0.6)	(2,3,4,6, 7,9;0.5)	(5,7,8,11, 14,17;0.9)	20
Demand	24	18	20	10	

Step 2:

Using ranking method **(3.1)** in the fuzzy transportation problem is converted into a transportation problem.

$$\begin{bmatrix} R(3,7,11\ 15,19,24:0,5) & R(13,18,23,28,33,40;0.7) & R(6,13,20,28,36,45;0.4) & R(15,20,25,31,38,45;0.8) \\ R(16,19,24,29,34,39;0.2) & R(3,5,7,9,10,12;0.5) & R(5,7,10,13,17,21;0.6) & R(20,23,26,30,35,40;0.4) \\ R(11,14,17,21,25,30;0.7) & R(7,9,11,14,18,22;0.6) & R(2,3,4,6,7,9;0.5) & R(5,7,8,11,14,17;0.9) \\ R(0,0,0,0,0,0) & R(0,0,0,0,0,0) & R(0,0,0,0,0,0) & R(0,0,0,0,0,0) \end{bmatrix}$$

$$R(\tilde{A}) = \frac{w}{2}[(a_6 - a_1) + 2(a_5 - a_2) + (a_4 - a_3)]$$

$$R(3,7,11,15,19,24;0.5) \quad = \frac{0.5}{2}[(24-3) + 2\times(19-7) + (15-11)] \quad = 12.25$$

Similarly $R(13,18,23,28,33,40;0.7) = 21.7$ $\quad R(6,13,20,28,36,45;0.4) = 18.6$

$R(15,20,25,31,38,45;0.8) = 28.6$ $\quad R(16,19,24,29,34,39;0.2) = 5.8$

$R(3,5,7,9,10,12;0.5) = 5.25$ $\quad R(5,7,10,13,17,21;0.6) = 11.7$

$R(20,23,26,30,35,40;0.4) = 9.6$ $\quad R(11,14,17,21,25,30;0.7) = 15.75$

$R(7,9,11,14,18,22;0.6) = 10.8$ $\quad R(2,3,4,6,7,9;0.5) = 4.25$

$R(5,7,8,11,14,17;0.9) = 13.05$ $\quad R(0,0,0,0,0,0;0) = 0$

$R(0,0,0,0,0,0;0) = 0$ $\quad R(0,0,0,0,0,0;0) = 0$

$R(0,0,0,0,0,0;0) = 0$

TABLE 6.2 TRANSPORTATION TABLE

Foundries / Machines	F_1	F_2	F_3	F_4	Supply
M_1	12.25	21.7	18.6	28.8	16
M_2	5.8	5.25	11.7	9.6	36
M_3	15.75	10.8	4.25	13.05	20
Demand	24	18	20	10	

Step 3:
Applying NWC method the initial basic feasible solution is obtained

TABLE 6.3 INITIAL BASIC FEASIBLE SOLUTION

Foundries / Machines	F_1	F_2	F_3	F_4	Supply
M_1	**16** 12.25	21.7	18.6	28.8	16
M_2	**8** 5.8	**18** 5.25	**10** 11.7	9.6	36
M_3	15.75	10.8	**10** 4.25	**10** 13.05	20
Demand	24	18	20	10	

Step 4:
The transportation cost according to the above route is given by

$Z = 12.25\times16 + 5.8\times8 + 5.25\times18 + 11.7\times 10 + 4.25 \times10 + 13.05\times10$

$Z = 626.9$

RESULT AND CONCLUSION:

The overall cost of the process is 626.9 which is the minimum cost. This cost is arrived when we supply 16 machines of M_1 and 8 machines of M_2 to foundries F_1 for a demand of 24 units. Similarly for a demand of 18 units for foundry F_2 is supplied with 18 machines of M_2. Then for a demand of 20 machines in foundry F_3 is supplied with 10 units of machine M_2 and with 10 units of machine M_3. Then for a demand of 10 units for foundry F_4 is supplied with 10 units of machine M_3.This is the optimal cost on consideration of various parameters.

In this paper we have proposed a method where a hexagonal fuzzy ranking is obtained using an area of trapezium. This method can be used in a Transportation problem where a number of parameters are involved and the fuzzy hexagonal values are converted into a crisp value and the solution is obtained as explained in the example.

Future Directions: This method can also be used easily for different industrial as well as social applications where the parameters to be analysed are more in number.

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