New Solitary Wave Solutions for Some Non-linear Partial Differential Equations

Anwar Ja'afar Mohamad Jawad

Computer Engineering Technique Department, Al -Rafidain University College, Baghdad, Iraq
anwar_jawad2001@yahoo.com

Abstract— **In this paper, the tanh and Sech function methods are proposed to establish a traveling wave solution for nonlinear partial differential equations. The two methods are used to obtain new solitary wave solutions for Drinfel'd–Sokolov–Wilson equation, Fornberg-Whitham equation, potential-TSF equation, Jimbo-Miwa equation, Modified Zakharov–Kuznetsov equation, and (2 + 1)-dimensional Konopelchenko-Dubrovsky equation. Methods have been successfully implemented to establish new solitary wave solutions for the nonlinear PDEs.**

Keywords: Nonlinear PDEs, Tanh function method, Sech function method, Drinfel'd–Sokolov–Wilson equation, Fornberg-Whitham equation, potential-TSF equation, Jimbo-Miwa equation, Modified Zakharov–Kuznetsov equation, and (2 + 1)-dimensional Konopelchenko-Dubrovsky equation

I. INTRODUCTION

Wave phenomena are observed in plasma, elastic media, optical fibers, fluid dynamics, etc. Constructing traveling wave solutions for nonlinear partial differential equations is an ongoing research. These traveling wave solutions when they exist can help one to well understand the mechanism of the complicated physical phenomena and dynamical processes modeled by these nonlinear partial differential equations. Many significant methods for obtaining exact solutions of nonlinear partial differential equations (NPDEs) have been presented [1-10].

This paper is motivated by the desire to find periodic wave solutions with the use of the Tanh and Sech functions. This means that the method will lead to a deeper and more comprehensive understanding of the complex structure of the nonlinear partial differential equations (NPDEs). On the one hand, to seek more formal solutions of NPDEs is needed from mathematical point of view

Six nonlinear wave equations were studied in this paper. These equations play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, geochemistry, and so on. These equations are in the following:

The Drinfel'd–Sokolov–Wilson equation (DSW)[11]:

$$u_t + p\, v\, v_x = 0\,;$$
$$v_t + q\, v_{xxx} + r\, u\, v_x + s\, u_x\, v = 0 \qquad (1)$$

The (1+1) dimensional Fornberg – Whitham equation[12]:

$$u_t - u_{xxt} + u_x - u u_{xxx} + u u_x - 3u_x u_{xx} = 0 \qquad (2)$$

The potential- TSF equation [13]:

$$u_{xxxz} + 4\, u_x\, u_{xz} + 2\, u_{xx}\, u_z - 4\, u_{xt} + 3\, u_{yy} = 0 \qquad (3)$$

the Jimbo-Miwa equation [14]:

$$u_{xxxy} + 3\, u_x\, u_{xy} + 3\, u_{xx}\, u_y + 2\, u_{yt} - 3\, u_{xz} = 0 \qquad (4)$$

the Modified Zakharov–Kuznetsov (mZK) equation [15]:

$$u_t + u^2\, u_x + (u_{xx} + u_{yy})_x = 0 \qquad (5)$$

the (2 + 1)-dimensional Konopelchenko-Dubrovsky equations[16]:

$$u_t - u_{xxx} - 6b\, u u_x + \frac{3\,a^2}{2}\, u^2\, u_x - 3\, v_y + 3av\;\; u_x = 0$$
$$u_y = v_x \qquad (6)$$

II. THE TWO METHODS METHODOLOGY

Consider the nonlinear partial differential equation in the form:

$$F\big(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{yy}, \dots \dots \dots \dots\big) = 0 \tag{7}$$

where u(x, y, t) is a traveling wave solution of nonlinear partial differential equation. We use the transformations $u(x, y, t) = f(\xi)$

Where:

$$\xi = x + y + \lambda t \tag{8}$$

where λ is the wave speed. This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = \lambda \frac{d}{d\xi}(.)\ ,\ \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.)\ , \frac{\partial}{\partial y}(.) = \frac{d}{d\xi}(.) \tag{9}$$

Using Eq. (8) to transfer the nonlinear partial differential equation Eq. (7) to nonlinear ordinary differential equation

$$Q(f, f', f'', f''', \dots \dots \dots \dots \dots .) = 0 \tag{10}$$

The ordinary differential equation (10) is then integrated as long as all terms contain derivatives, where we neglect the integration constants.

A. Topological Soliton Solution by Tanh Method

The solutions of many nonlinear equations can be expressed in the form:

$$f(\xi) = \alpha \tanh^{\beta}(\mu\xi)\ \ ,\ \ |\xi| \le \frac{\pi}{2\mu} \tag{11}$$

with the derivatives of Eq. (11) :

$$f'(\xi) = \alpha\, \beta\, \mu\, \left[\tanh^{\beta-1}(\mu\xi) - \tanh^{\beta+1}(\mu\xi))\right]$$

$$f''(\xi) = \alpha\, \beta\, \mu^2\, \left[(\beta - 1)\tanh^{\beta-2}(\mu\xi) - 2\beta \tanh^{\beta}(\mu\xi) + (\beta + 1)\tanh^{\beta+2}(\mu\xi)\right]$$

$$f'''(\xi) = \alpha\beta\, \mu^3 \begin{bmatrix} (\beta-1)(\beta-2)\tanh^{\beta-3}(\mu\xi)\ -\{(\beta-1)(\beta-2)+2\beta\}\tanh^{\beta-1}(\mu\xi) \\ +\{(\beta+1)(\beta+2)+2\beta\}\tanh^{\beta+1}(\mu\xi) - (\beta+1)(\beta+2)\tanh^{\beta+3}(\mu\xi) \end{bmatrix} \tag{12}$$

Where α, μ , and β are parameters to be determined, μ is the wave number. We substitute Eq. (11) into the reduced equation Eq. (10), balance the terms of the tanh functions and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $\tanh^{k}(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknown's α , μ and β, and solve the subsequent system.

B. Non-Topological Soliton Solution by Sech Method

The solutions of many nonlinear equations can be expressed in the form:

$$f(\xi) = \alpha\, \mathrm{sech}^{\beta}(\mu\xi) \tag{13}$$

We use

$$f'(\xi) = -\,\alpha\, \beta\, \mu\ \ \mathrm{sech}^{\beta}(\mu\xi)\ \tanh(\mu\xi)$$

$$f''(\xi) = -\,\alpha\, \beta\, \mu^2 \left[(\beta+1)\mathrm{sech}^{\beta+2}(\mu\xi) - \beta\, \mathrm{sech}^{\beta}(\mu\xi)\right]$$

$$f'''(\xi) = \ \alpha\, \beta\, \mu^3 \left[(\beta+1)(\beta+2)\, \mathrm{sech}^{\beta+2}(\mu\xi) - \beta^2\, \mathrm{sech}^{\beta}(\mu\xi)\right] \tanh(\mu\xi) \tag{14}$$

and so on. We substitute (14) into the reduced equation (10), balance the terms of the Sech functions are used, and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $\mathrm{sech}^{k}(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknown's α , μ and β, and solve the subsequent system.

III. APPLICATIONS:

A. The Drinfel'd–Sokolov–Wilson equation (DSW):

Now we will bring to find exact solution and then the solitary wave solution to the DSW equation in the form [11].

$$u_t + p\, v\, v_x \ = 0\, ;$$

$$v_t + q\, v_{xxx} + r\, u\, v_x + s\, u_x\, v \;= 0 \tag{15}$$

where p, q, r and s are nonzero parameters.

Khan et al [11] studied this equation and the modified simple equation (MSE) method was implemented to find the exact solutions.

Now let us suppose that the traveling wave transformation equation be

$$u(\xi) = u\,(x,t), \qquad v(\xi) = v\,(x,t)\,, \quad \xi = x + \lambda\, t \tag{16}$$

where λ is real constant. The Eq. (16) reduces Eq. (15) into the following ODEs

$$\lambda\, u' + P\, v\, v' \;= 0\,; \tag{17}$$

$$\lambda\, v' + q\, v''' + r\, u\, v' + s\, u'\, v = 0 \tag{18}$$

By integrating Eq. (17) with respect to ξ , and neglecting the constant of integration, we obtain:

$$u = -P\, \frac{v^2}{2\,\lambda} \tag{19}$$

Substituting Eq. (19) into Eq. (18), we obtain

$$2\,\lambda^2\, v' + 2\,\lambda\, q\, v''' - p\,[\; r + 2\, s\,]\;\; v^2\, v' = 0 \tag{20}$$

Integrating Eq. (18) with respect to ξ choosing constant of integration to zero, we obtain:

$$2\,\lambda^2\, v + 2\,\lambda\, q\, v'' - \frac{p}{3}\;[\; r + 2\, s\,]\;\; v^3 = 0 \tag{21}$$

For the Topological Soliton Solution by Tanh Method from (12), Equation (21) becomes

$$\begin{bmatrix} 2\,\lambda^2\, \alpha \tanh^{\beta}(\mu\xi) + \\ 2\,\lambda\, q\, \alpha\, \beta\, \mu^2\, \left[(\beta-1)\tanh^{\beta-2}(\mu\xi) - 2\beta\tanh^{\beta}(\mu\xi) + (\beta+1)\tanh^{\beta+2}(\mu\xi)\right] \\ -\frac{p}{3}\;[\; r + 2\, s\,]\, \alpha^3 \tanh^{3\beta}(\mu\xi) = 0 \end{bmatrix} \tag{22}$$

Equating the exponents of tanh terms $3\beta = \beta + 2$, then $\beta = 1$. Substituting for $\beta = 1$ in Eq. (22) to get the following system of equations:

$$\begin{gathered} \lambda\; - 2\;\, q\;\; \mu^2 = 0 \\ 4\,\lambda\, q\;\; \mu^2 - \frac{p}{3}\;[\; r + 2\, s\,]\, \alpha^2 = 0 \end{gathered} \tag{23}$$

Solving system (23) to get

$$\mu = \mp\sqrt{\frac{\lambda}{2\,q}} \qquad \alpha = \mp\sqrt{\frac{6}{p\,[\; r+2\,s\,]}}\;\; \lambda \tag{24}$$

Then:

$$v(x,t) = \mp\sqrt{\frac{6}{p\,[\; r+2\,s\,]}}\;\; \lambda\;\;\; \tanh\left(\sqrt{\frac{\lambda}{2\,q}}\;(x + \lambda\, t)\right) \tag{25}$$

and

$$u(x,t) = -\,\frac{3\,\lambda}{r+2\,s}\, \tanh^2\left(\sqrt{\frac{\lambda}{2\,q}}\;(x + \lambda\, t)\right) \tag{26}$$

for λ = -1, p= 3, q = -2, r =2, s= 1

$$v(x,t) = \mp\sqrt{\frac{1}{2}}\;\;\; \tanh\left(\frac{(x-t)}{2}\right) \tag{27}$$

and

$$u(x,t) = \;\frac{3}{4}\, \tanh^2\left(\frac{(x-t)}{2}\right) \tag{28}$$

Figures.(1) and (2) show the shape of the exact Kink-type solution $v(x,t)$ and $u(x,t)$ in Eq.(27) and (28) respectively for $-10 \le x \le\; 0,\;\; 0 \le t\; \le 10$.

We observe that $u(x,t) = u(x+\lambda t)$ and $v(x,t) = v(x+\lambda t)$ which means that for negative value of wave speed (here $\lambda = -1$) the disturbance moving in the positive x-direction. If we take $\lambda > 0$ the propagation will be in the negative x-direction

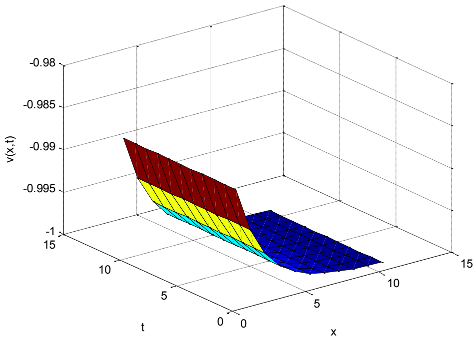

Fig.(1) Solitary wave solution $v(x,t)$ in Eq.(27) for $-10 \leq x,\ t \leq 10$.

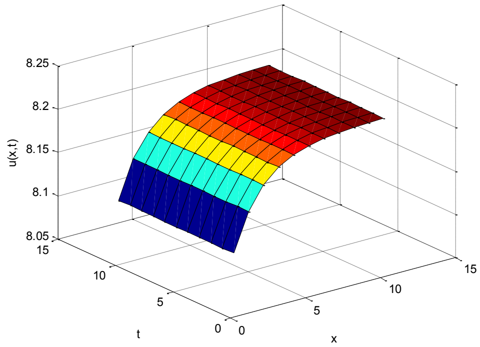

Fig.(2) Solitary wave solution $u(x,t)$ in Eq.(28) for $-10 \leq x,\ t \leq 10$.

B. The (1+1) dimensional Fornberg – Whitham equation

The (1+1) dimensional Fornberg – Whitham equation is on the form[12].

$$u_t - u_{xxt} + u_x - uu_{xxx} + uu_x - 3u_x u_{xx} = 0 \tag{29}$$

Shehata et al [12] studied this equation and used G'/G method.

Assume the traveling wave transformation

$$U(\xi) = u\,(x,t), \qquad \xi = x + \lambda\, t \tag{30}$$

That permits us converting Eq.(29) to the following ODE:

$$(\lambda+1)U' - \lambda\, U''' - UU''' + UU' - 3\, U'U'' = 0 \tag{31}$$

Eq.(31) can be arranged an written as

$$(\lambda+1)U' - \lambda\, U''' - UU''' - U'U'' + UU' - 2\, U'U'' = 0 \tag{32}$$

Then Eq.(32) is:

$$(\lambda+1)U' - \lambda\, U''' - [\,UU'']' + \tfrac{1}{2}(U^2)' - \left(U'^2\right)' = 0 \tag{33}$$

Integrating Eq.(33) once with zero constants

$$(\lambda+1)U - \lambda\, U'' - UU'' + \tfrac{1}{2}U^2 - U'^2 = 0 \tag{34}$$

I). Non-Topological Soliton Solution by Sech Method

Seeking the non-topological solution as in Eq.(13), Eq.(34) becomes

$$(\lambda+1)\,\mathrm{sech}^{\beta}(\mu\xi)+\lambda\,\beta\,\mu^2\left[(\beta+1)\mathrm{sech}^{\beta+2}(\mu\xi)-\beta\,\mathrm{sech}^{\beta}(\mu\xi)\right]$$
$$+\alpha\,\beta\,\mu^2\left[(\beta+1)\mathrm{sech}^{2\beta+2}(\mu\xi)-\beta\,\mathrm{sech}^{2\beta}(\mu\xi)\right]$$
$$+\frac{1}{2}\alpha\,\mathrm{sech}^{2\beta}(\mu\xi)-\alpha\,\beta^2\mu^2\,\mathrm{sech}^{2\beta}(\mu\xi)+\alpha\,\beta^2\mu^2\,\mathrm{sech}^{2\beta+2}(\mu\xi)=0 \tag{35}$$

Equating the exponents of sech of two terms $2\beta=\beta+2$ then $\beta=2$. Equating the coefficients of the same powers of Eq.(35). System of equations is established:

$$(\lambda+1)-4\lambda\mu^2=0$$
$$10\,\alpha\,\mu^2=0$$
$$12\,\lambda\,\mu^2+\alpha-16\,\alpha\,\mu^2=0 \tag{36}$$

Solving system (36), to get:

$$\lambda=\frac{1}{[4\mu^2-1]}\;,\;\alpha=\frac{12\mu^2}{[1-4\mu^2][1-16\mu^2]} \tag{37}$$

Then

$$u(x,t)=\frac{12\mu^2}{[1-4\mu^2][1-16\mu^2]}\;\mathrm{sech}^2\left[\mu\left(x+\frac{1}{[4\mu^2-1]}t\right)\right] \tag{38}$$

or

$$u(x,t)=\frac{12\mu^2}{[1-4\mu^2][1-16\mu^2]}\left\{1-\tanh^2\left[\mu\left(x+\frac{1}{[4\mu^2-1]}t\right)\right]\right\} \tag{39}$$

Such that:

$$\mu^2\in R\left|\mp\frac{1}{4}\;,\;\mp\frac{1}{2}\right. \tag{40}$$

$$u(x,t)=\frac{4}{15}\left\{1-\tanh^2\left[\left(x+\frac{1}{3}t\right)\right]\right\} \tag{41}$$

Fig.(3) presents the Solitary solution u(x,t) for $-5\le x\le 5\,,0\le t\le 5$

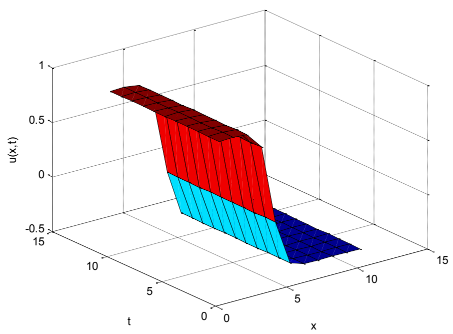

Fig.(3) the Solitary solution u(x,t) for $-5\le x\le 5\,,0\le t\le 5$

II). Topological Soliton Solution by Tanh Method

For the Topological Solution by Tanh Method from (11), Equation (31) becomes

$$(\lambda+1)\tanh^{\beta}(\mu\xi)$$
$$-\lambda\,\beta\,\mu^2\left[(\beta-1)\tanh^{\beta-2}(\mu\xi)-2\beta\tanh^{\beta}(\mu\xi)+(\beta+1)\tanh^{\beta+2}(\mu\xi)\right]$$
$$-\alpha\,\beta\,\mu^2\left[(\beta-1)\tanh^{2\beta-2}(\mu\xi)-2\beta\tanh^{2\beta}(\mu\xi)+(\beta+1)\tanh^{2\beta+2}(\mu\xi)\right]$$
$$+\frac{1}{2}\alpha\tanh^{2\beta}(\mu\xi)-\alpha\,\beta^2\mu^2\left[\tanh^{2\beta-2}(\mu\xi)-2\tanh^{2\beta}(\mu\xi)+\tanh^{2\beta+2}(\mu\xi))\right]=0 \tag{42}$$

Equating the exponents of two tanh terms $2\beta=\beta+2$ then $\beta=2$. Equating the coefficients of the same powers of Eq.(42). System of equations is:

$$\tanh^2(\mu\xi) \ : \qquad (\lambda+1)\ +4\lambda\ \ \beta\,\mu^2\ =0$$
$$\tanh^4(\mu\xi) \ : \qquad \alpha\ +32\ \ \alpha\ \ \mu^2-12\lambda\,\mu^2=0 \tag{43}$$

Solving system (43) to get:

$$\alpha\ =-\frac{12\ \mu^2}{256\ \ \mu^4+40\ \mu^2+1}\ ,\qquad \lambda\ =\ -\frac{32\ \mu^2+1}{256\ \ \mu^4+40\ \mu^2+1} \tag{44}$$

then:

$$u(x,t)=\ -\frac{12\ \mu^2}{256\ \ \mu^4+40\ \mu^2+1}\ \mathrm{sech}^2\left[\ \mu\left(x-\frac{32\ \mu^2+1}{256\ \ \mu^4+40\ \mu^2+1}\ t\right)\right] \tag{45}$$

for $\mu=1$

$$u(x,t)=\ -\frac{4}{99}\ \left\{1-\tanh^2\left(\ x-\frac{1}{9}\ t\right)\right\} \tag{46}$$

Figure (4) illustrates the solution $u(x,t)$ from Eq.(46) for $-5\le x\le 5\,, 0\le t\le 5$

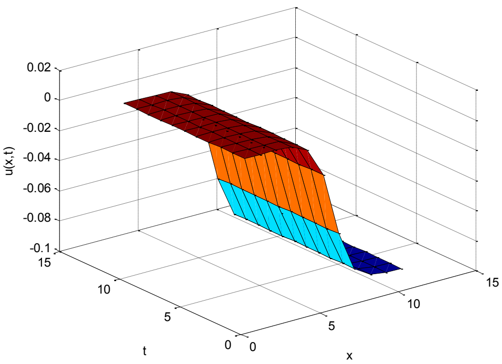

Fig.(4) the Solitary solution u(x,t) for $-5\le x\le 5\,, 0\le t\le 5$

C. The potential- TSF equation

Consider the potential- TSF equation in the form:

$$u_{xxxz}+4\ u_x\ u_{xz}+2\ u_{xx}\ u_z-4\ u_{xt}+3\ u_{yy}=0 \tag{47}$$

This equation was studied by Borhanifar et al [14] by applying the exp-function method.

The traveling wave transformation

$$U(\xi)=u\ (x,y,z,t),\qquad \xi=k\,x+w\,y+\sigma\,z+\lambda\,t+\ \theta_0 \tag{48}$$

Convert Eq.(47) to the following ODE:

$$k^3\ \sigma\ U''''+6\ k^2\ \sigma\ U'U''\ +(3\ w^2\ -4\ k\ \lambda)\ U''=0 \tag{49}$$

Eq.(47) can be written as:

$$k^3\ \sigma\ U''''+3\ k^2\ \sigma\left(U'^2\right)'\ +(3\ w^2\ -4\ k\ \lambda)\ U''=0 \tag{50}$$

Integrating Eq.(50) once with zero constant

$$k^3\ \sigma\ U'''+3\ k^2\ \sigma\ U'^2\ +(3\ w^2\ -4\ k\ \lambda)\ U'=0 \tag{51}$$

Seeking the topological solution as in Eq.(11), Eq.(51) becomes

$$k^3\ \sigma\ \alpha\beta\ \mu^3\begin{bmatrix}(\beta-1)(\beta-2)\ \tanh^{\beta-3}(\mu\xi)\ -\{(\beta-1)(\beta-2)+2\beta\}\tanh^{\beta-1}(\mu\xi)\\ +\{(\beta+1)(\beta+2)+2\beta\}\ \tanh^{\beta+1}(\mu\xi)-(\beta+1)(\beta+2)\ \tanh^{\beta+3}(\mu\xi)\end{bmatrix}+$$
$$3\ k^2\ \sigma\ \alpha^2\ \beta^2\ \mu^2\ \left[\tanh^{2\beta-2}(\mu\xi)-2\ \tanh^{2\beta}\ +\tanh^{2\beta+2}(\mu\xi))\right]\ +$$
$$(3\ w^2\ -4\ k\ \lambda)\ \alpha\ \beta\ \mu\ \left[\tanh^{\beta-1}(\mu\xi)-\tanh^{\beta+1}(\mu\xi))\right]=0 \tag{52}$$

Equating the exponents of two tanh terms $2\beta+2=\beta+3$ and $2\beta-2=\beta-1$ and $2\beta=\beta+1$ then $\beta=1$. Equating the coefficients of the same powers of Eq.(52). System of equations is established:

$$-2\,k^3\,\sigma\;\;\mu^2\;+3\,k^2\,\sigma\,\alpha\;\;\mu\;\;+(3\,w^2\;-4\,k\,\lambda)=0$$
$$8\,k^3\,\sigma\;\;\mu^2\;-6\,k^2\,\sigma\,\alpha\;\;\mu-(3\,w^2\;-4\,k\,\lambda)\;=0$$
$$-6\,k^3\,\sigma\;\;\mu^2\;+3\,k^2\,\sigma\,\alpha\;\;\mu\;\;=0 \tag{53}$$

Solving system (53), to get:

$$\alpha=\;\mp\sqrt{\frac{4k\,\lambda-3\,w^2}{k\,\sigma}}\quad,\quad\mu=\mp\sqrt{\frac{4k\,\lambda-3\,w^2}{4\;k^3\,\sigma}} \tag{54}$$

Then

$$u(x,y,z,t)=\mp\sqrt{\frac{4k\,\lambda-3\,w^2}{k\,\sigma}}\;\;\tanh\left[\;\sqrt{\frac{4k\,\lambda-3\,w^2}{4\;k^3\,\sigma}}\;(k\,x+w\,y+\sigma\,z+\lambda\,t+\;\theta_0)\right] \tag{55}$$

Such that:

$$4k\,\lambda-3\,w^2>0 \tag{56}$$

for $k=w=\sigma=\lambda=1,\; z=y=1,\theta_0=0$

$$u(x,1,1,t)=\mp\tanh\left[\tfrac{1}{2}\,(\,x+\;t)+1\right] \tag{57}$$

Fig.(5) illustrates the Solitary solution u(x, t) in Eq.(57) for $-5\leq x\leq 5\,,\;\;0\leq t\leq 5,\;\;y=z=1$

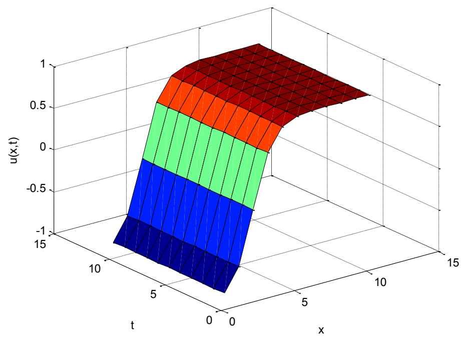

Fig.(5) the Solitary solution *u(x, t) in Eq.(57) for* $-5\leq x\leq 5\,,\;\;0\leq t\leq 5,\;\;y=z=1$

D. (3+1)-dimensional Jimbo-Miwa Equation

Let us consider the Jimbo-Miwa equation in the form:

$$u_{xxxy}+3\,u_x\,u_{xy}+3\,u_{xx}\,u_y+2\,u_{yt}-3\,u_{xz}=0 \tag{58}$$

This equation was studied by Anwar [13] by using Tanh method. Borhanifar [14] studied the equation by applying the exp-function method.

By using the same traveling wave transformations in (48) the nonlinear Eq.(58) is carried to an ordinary differential equation

$$k^3\,w\,U''''+6\,k^2\,w\,U'U''\;+(2\,\lambda\,w\;-3\,k\,\sigma)\,U''=0 \tag{59}$$

Eq.(59) can be written as:

$$k^3\,w\,U''''+3\,k^2\,w\left[U'^2\right]'\;+(2\,\lambda\,w\;-3\,k\,\sigma)\,U''=0 \tag{60}$$

Integrating Eq.(60) once with zero constant

$$k^3\,w\,U'''+3\,k^2\,w\,U'^2\;+(2\,\lambda\,w\;-3\,k\,\sigma)\,U'=0 \tag{61}$$

Seeking the topological solution as in Eq.(11), Eq.(61) becomes

$$k^3\,w\,\alpha\,\beta\,\mu^3\begin{bmatrix}(\beta-1)(\beta-2)\tanh^{\beta-3}(\mu\xi)\;-\{(\beta-1)(\beta-2)+2\beta\}\tanh^{\beta-1}(\mu\xi)\\+\{(\beta+1)(\beta+2)+2\beta\}\tanh^{\beta+1}(\mu\xi)-(\beta+1)(\beta+2)\tanh^{\beta+3}(\mu\xi)\end{bmatrix}+$$
$$3\,k^2\,w\,\alpha^2\,\beta^2\,\mu^2\,\left[\tanh^{2\beta-2}(\mu\xi)-2\tanh^{2\beta}\;+\tanh^{2\beta+2}(\mu\xi))\right]+$$
$$(2\lambda w\;-3\,k\,\sigma)\,\alpha\,\beta\,\mu\,\left[\tanh^{\beta-1}(\mu\xi)-\tanh^{\beta+1}(\mu\xi))\right]=0 \tag{62}$$

Equating the exponents of two tanh terms $2\beta+2=\beta+3$ and $2\beta-2=\beta-1$ and $2\beta=\beta+1$ then $\beta=1$. Equating the coefficients of the same powers of Eq.(62). System of equations is established:

$$-2\,k^3\,w\,\mu^2+3\,k^2\,w\,\alpha\,\mu+(2\lambda w-3\,k\,\sigma)=0$$
$$8\,k^3\,w\,\mu^2-6\,k^2\,w\,\alpha\,\mu-(2\lambda w-3\,k\,\sigma)=0$$
$$-6\,k^3\,w\,\mu^2+3\,k^2\,w\,\alpha\,\mu=0 \tag{63}$$

Solving system (63), to get:

$$\mu=\mp\sqrt{\frac{3\,k\,\sigma-2\lambda w}{4\,k^3\,w}}\ ,\ \alpha=\mp\sqrt{\frac{3\,k\,\sigma-2\lambda w}{k\,w}} \tag{64}$$

Then

$$u(x,y,z,t)=\mp\sqrt{\frac{3\,k\,\sigma-2\lambda w}{k\,w}}\ \tanh\left[\sqrt{\frac{3\,k\,\sigma-2\lambda w}{4\,k^3\,w}}\,(k\,x+w\,y+\sigma\,z+\lambda\,t+\theta_0)\right] \tag{65}$$

Such that:

$$3\,k\,\sigma-2\lambda w>0 \tag{66}$$

for $k=w=\sigma=\lambda=1,\ z=y=1,\theta_0=0$

$$u(x,1,1,t)=\mp\tanh\left[\tfrac{1}{2}\,(x+t)+1\right] \tag{67}$$

E. Modified Zakharov–Kuznetsov (mZK)

Let us consider the Modified Zakharov–Kuznetsov (mZK) equation in the following

$$u_t+u^2\,u_x+(u_{xx}+u_{yy})_x=0 \tag{68}$$

This equation was studied by Tascan et al. [15] by the first integral method.

By using tanh method and using the traveling wave transformations in (48) the nonlinear Eq.(68) is carried to an ordinary differential equation

$$\lambda\,U'+k\,U^2U'+(k^2+w^2)\,U'''=0 \tag{69}$$

Integrating Eq.(69) once with zero constant

$$\lambda\,U+\frac{k}{3}\,U^3+(k^2+w^2)\,U''=0 \tag{70}$$

Seeking the topological solution as in Eq.(11), Eq.(70) becomes

$$\lambda\,\alpha\,\tanh^{\beta}(\mu\xi)+\frac{k}{3}\,\alpha^3\,\tanh^{3\beta}(\mu\xi)+(k^2+w^2)\alpha\,\beta\,\mu^2\left[(\beta-1)\tanh^{\beta-2}(\mu\xi)-2\beta\tanh^{\beta}(\mu\xi)+(\beta+1)\tanh^{\beta+2}(\mu\xi)\right]=0 \tag{71}$$

Equating the exponents of two tanh terms $\beta+2=3\beta$ and $\beta=1$. Equating the coefficients of the same powers of Eq.(71). System of equations is established:

$$\lambda-2(k^2+w^2)\,\mu^2=0$$
$$\frac{k}{3}\,\alpha^2+2(k^2+w^2)\,\mu^2=0 \tag{72}$$

Solving system (72), to get:

$$\lambda=2(k^2+w^2)\,\mu^2\ ,\quad \alpha=\mp i\sqrt{\frac{6(k^2+w^2)}{k}}\,\mu \tag{73}$$

Then

$$u(x,t)=\mp i\sqrt{\frac{6\,(k^2+w^2)}{k}}\,\mu\ \tanh[\ \mu\,(k\,x+w\,y+2(k^2+w^2)\,\mu^2\,t+\theta_0)] \tag{74}$$

or

$$u(x,y,t)=\mp\sqrt{\frac{6\,(k^2+w^2)}{k}}\,\mu\tan[i\,\mu\,(k\,x+w\,y+2(k^2+w^2)\,\mu^2\,t+\theta_0)] \tag{75}$$

Such that:

$k > 0$

for $k = w = \mu = 1,\ y = 0, \theta_0 = 0$

$$u(x, 0, t) = \mp 2\sqrt{3} \ \tan[\, i\, (x + 4\, t)] \tag{76}$$

Fig.(6) presents the Solitary solution u(x, t) in Eq.(76) for $-5 \le x \le 5\,,\ \ 0 \le t \le 5,\ y = 0$

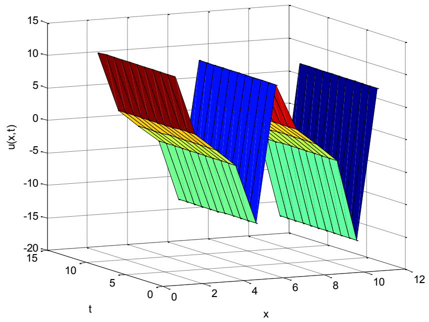

Fig.(6) the Solitary solution u(x, t) in Eq.(78) for $-5 \le x \le 5\,,\ \ 0 \le t \le 5,\ y = 0$

F. (2 + 1)-dimensional Konopelchenko-Dubrovsky equation

In this section we consider the (2 + 1)-dimensional Konopelchenko-Dubrovsky system of equation in the form

$$u_t - u_{xxx} - 6b\, uu_x + \frac{3\,a^2}{2}\, u^2\, u_x - 3\, v_y + 3av\ u_x = 0$$

$$u_y = v_x \tag{77}$$

This equation was studied by Taghizade et al [16] by the G'/G -Expansion Method.

By using the traveling wave transformations in (48) then Eqs.(77) are carried to the following ordinary differential equations

$$\lambda\, U' - k^3\, U''' - 6b\, k\, UU' + \frac{3\,a^2}{2}\, k\, U^2 U' - 3\, w\, V' + 3a\, k\, VU' = 0 \tag{78}$$

$$wU' = k\, V' \tag{79}$$

Integrating Eq.(79) once with zero constants:

$$wU = k\, V \tag{80}$$

Substituting for V and V' from Eq.(79) and (80) in Eq.(78), then Eq.(78) becomes:

$$\lambda\, k\, U' - k^4\, U''' - 6b\, k^2\, UU' + \frac{3\,a^2}{2}\, k^2\, U^2 U' - 3\, w\, wU' + 3ak\, wUU' = 0 \tag{81}$$

Integrating Eq.(81) once with zero constant

$$[\,\lambda\, k - 3\, w^2\,]\, U + \frac{a^2}{2}\, k^2\, U^3\ + [\tfrac{3}{2} ak\, w - 3\, b\, k^2]\, U^2 - k^4\, U'' = 0 \tag{82}$$

Seeking the topological solution as in Eq.(13), Eq.(82) becomes

$$\begin{gathered}[\,\lambda\, k - 3\, w^2\,]\, \mathrm{sech}^{\beta}(\mu\xi) + \frac{a^2}{2}\, k^2\, \alpha^2\, \mathrm{sech}^{3\beta}(\mu\xi)\ + \\ \left[\tfrac{3}{2} ak\, w - 3\, b\, k^2\right] \alpha\, \mathrm{sech}^{2\beta}(\mu\xi) + \\ k^4\, \beta\, \mu^2\, \left[(\beta + 1)\mathrm{sech}^{\beta+2}\,(\mu\xi) - \beta\, \mathrm{sech}^{\beta}\,(\mu\xi)\right] = 0\end{gathered} \tag{83}$$

Equating the exponents of two tanh terms $\beta + 2 = 3\beta$ and $\beta = 1$. Equating the coefficients of the same powers of Eq.(83). System of equations is established:

$$[\,\lambda\, k - 3\, w^2\,] - k^4\ \mu^2\ \ = 0$$

$$\frac{3}{2} ak\, w - 3\, b\, k^2 = 0$$

$$\frac{a^2}{2}\ \alpha^2 + 2\, k^2\, \mu^2 = 0 \tag{84}$$

Solving system (84) to get:

$$\mu = \mp\sqrt{\frac{a^2\,\lambda - 12b^2\,k}{k^3\,a^2}} \quad , = \frac{2\,b\,k}{a}, \quad \alpha = \mp\frac{2}{a^2}\sqrt{12b^2 - \frac{a^2\,\lambda}{k}} \tag{85}$$

Then

$$u(x,t) = \mp\frac{2}{a^2}\sqrt{\frac{12k\,b^2 - a^2\,\lambda}{k}}\ \mathrm{sech}\left[\frac{i}{k\,a}\sqrt{\frac{12k\,b^2 - a^2\,\lambda}{k}}\,(\,k\,x + \frac{2\,b\,k}{a}\,y + \lambda\,t + \theta_0)\right] \tag{86}$$

or

$$u(x,y,t) = \mp i\,\frac{2}{a^2}\sqrt{\frac{12k\,b^2 - a^2\,\lambda}{k}}\ \sec\left[\frac{1}{k\,a}\sqrt{\frac{12k\,b^2 - a^2\,\lambda}{k}}\,(\,k\,x + \frac{2\,b\,k}{a}\,y + \lambda\,t + \theta_0)\right] \tag{87}$$

and

$$v(x,y,t) = \mp i\,\frac{4\,b}{a^3}\sqrt{\frac{12k\,b^2 - a^2\,\lambda}{k}}\ \sec\left[\frac{1}{k\,a}\sqrt{\frac{12k\,b^2 - a^2\,\lambda}{k}}\,(\,k\,x + \frac{2\,b\,k}{a}\,y + \lambda\,t + \theta_0)\right] \tag{88}$$

such that:

$$\frac{12k\,b^2 - a^2\,\lambda}{k} > 0 \tag{89}$$

For $a = b = k = \lambda = 1, \theta_0 = 0$

$$u(x,1,t) = \mp i\,2\,\sqrt{11}\,\sec\left[\sqrt{11}\,(\,x + 2y + t)\right] \tag{90}$$

and

$$v(x,1,t) = \mp i\,4\,\sqrt{11}\,\sec\left[\sqrt{11}\,(\,x + 2y + t)\right] \tag{91}$$

Fig.(7) presents the Solitary solution u(x, t) in Eq.(90) for $-5 \le x \le 5\,,\ \ 0 \le t \le 5,\ \ y = 1$.

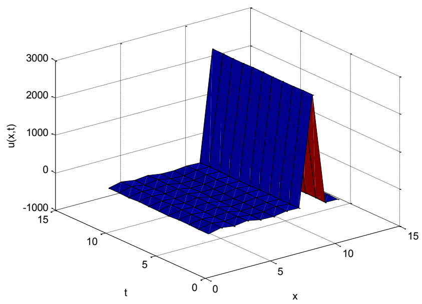

Fig.(7) Solitary solution u(x, t) in Eq.(94) for $-5 \le x \le 5\,,\ \ 0 \le t \le 5,\ \ y = 1$

IV. Conclusions

In this paper, the tanh and Sech function method has been successfully implemented to establish new solitary wave solutions for six nonlinear wave equations, namely, Drinfel'd–Sokolov–Wilson equation, Fornberg-Whitham equation, potential-TSF equation, Jimbo-Miwa equation, Modified Zakharov–Kuznetsov equation, and (2 + 1)-dimensional Konopelchenko-Dubrovsky equation. New solitary wave solutions for the nonlinear PDEs were established. The two methods can be extended to solve the problems of nonlinear partial differential equations.

References

[1] Hirota, R., "Exact solutions of the KdV equation for multiple collisions of solitons". Phys. Rev. Lett. 27: (1971)1192-1199.

[2] Kimiaki KONNO and Miki W ADAT, "Simple derivation of Backlund transformation from Riccati form of inverse method". Progress of Theoretical Physics, Vol. 53, (6), (1975) 1652-1956.

[3] AJM Jawad " Soliton solutions for Nonlinear systems (2+ 1)-Dimensional Equations" - IOSR Journal of Mathematics (IOSRJM), (2012), 2278-5728.

[4] AJ Mohamad-Jawad, MD Petković, A Biswas "Applications of He's principles to partial differential equations", Applied Mathematics and Computation 217 (16), 7039-7047

[5] Fu, Z.T., S.K. Liu, S.D. Liu and Q. Zhao, "New Jacobi elliptic function expansion and new periodic solutions of nonlinear wave equations". Phys. Lett. A., 290 (2001) 72-76.

[6] AJM Jawad " Soliton solutions for the Boussinesq equations" Journal of Mathematical and Computational Science, 3 (1),(2013), 254.

[7] Shen, Y. and N. Cao, "The double F-expansion approach and novel nonlinear wave solutions of soliton equation in (2+1) dimension". Applied Math. and Computation, 198: (2008), 683-690.

[8] AJM Jawad , " The Sine-Cosine Function Method for the Exact Solutions of Nonlinear Partial Differential Equations" Ijrras journal 13 (1), (2012).

[9] He, J. and M.A. Abdou, "New periodic solution for nonlinear evolution equations using Exp-function method". Choas Soliton and Fractals, 34: (2007),1421-1429.

[10] Peng, Y.Z., "Exact solutions for some nonlinear partial differential equations". Phys. Lett. A., 314: (2003), 401-408.

[11] Kamruzzaman Khan, M. Ali Akbar, Md. Nur Alam, "Traveling wave solutions of the nonlinear Drinfel'd–Sokolov–Wilson equation and modified Benjamin–Bona–Mahony equations", Journal of the Egyptian Mathematical Society (2013) 21, 233–240.

[12] Shehata A. R., Zayed E.M.E. and Gepree K. A., "Exact Solutions for Some Nonlinear Partial Differential Equations in Mathematical Physics", Journal of Information and Computing Science Vol. 6, No. 2,(2011), pp. 129-142

[13] AJ Mohamad-Jawad " Tanh Method For Solutions Of Non-linear Evolution Equations" ,Journal of Basrah Researches ((Sciences)) volume 37 (4), (2011).

[14] Borhanifar and Kabir M., "Soliton and Periodic Solutions for (3+1)-Dimensional Nonlinear Evolution Equations by Exp-function Method", Applications and Applied Mathematics Vol. 5, Issue 1 (2010), pp. 59 – 69.

[15] Tascan F, Bekir A, Koparan M. "Travelling wave solutions of nonlinear evolutions by using the first integral method". Commun Nonlinear Sci Numer Simul 14, (2009), 1810.

[16] Taghizade N., Neirameh A., "Some Applications of the G'/G -Expansion Method for Solving the Nonlinear Partial Differential Equations in Mathematical Physics", Applications and Applied Mathematics journal Vol. 4, Issue 2 (2009), pp. 290– 300.