Nanofluid Effects on the Flow with Suction or Injection Over a Moving Surface and Heat Transfer in Boundary Layer

Mohammad Mehdi Keshtkar\(^1\) and AmirTabatabaei mirhossyni \(^2\)

\(^1\) Assistant Professor, Department of Mechanical Engineering, \(^2\) MSc. student, Department of Mechanical Engineering, Islamic Azad University Kerman Branch, Iran

Abstract— In this study effect of boundary layer flow over flat plate in a nanofluid with two dimensional boundary layer. Uniform suction and injection on the flow field and heat transfer is analyzed. The nonlinear partial differential equation reduced by similarity transformation to the non-coupled nonlinear ordinary differential equations. Numerical shooting technique with a fourth-order Runge-Kutta scheme had used to obtain the solution of the boundary value problem. Using different types of nanoparticles as Cu (cuprum), Al\(_2\)O\(_3\) (aluminum) and TiO\(_2\) (titanium). The result of injection accelerates boundary layer separation.

Keywords-Nanofluid; Boundary layer injection; suction; Heat transfer.

I. INTRODUCTION

Small solid particles in fluids improving the heat transfer of fluids, this new kind of fluids named nanofluid and was introduced by choi in 1995 \([1]\). Nanofluid consists mixture of liquid like water and solid nanoparticles with high conductivity and low volume fraction. Enormous heat transfer can be results. This Nano fluid usually are very stable and without additional problems like erosion, pressure drop and non-Newtonian behavior. The Argonne National Laboratory has studied to use nanoparticles around a decade ago \([2, 3]\). Nanoparticles can change heat transfer and thermal properties. Therefore, it is very useful to enhance heat transfer in separated regions by using nanofluids. Such enhancement is accomplished by increasing the value of convective heat transfer coefficient (or Nusselt number) in separated flows. Suction or injection has affected the heat transfer rate at the surface. For example suction help to increase the skin fraction and heat transfer coefficients but injection tends to decrease that \([4]\). Injection of fluid through a porous bounding heated or cooled surface is usually interest in practical application including film cooling, control of boundary layer etc. This can lead to enhanced heating or cooling of the system and can help to delay the transition from laminar flow \([5]\).

II. DESCRIPTION OF PROBLEM AND FORMULATION

Consider the boundary layer flow of a viscous and incompressible nanofluid on a moving semi-infinite permeable flat plate. nanofluids based is water and nanoparticles are different types of Cu, Al\(_2\)O\(_3\) and TiO\(_2\). The flow is assumed laminar with no slip between them. The plate moves with a constant velocity \( \bar{u}_w = \lambda U \), where \( \lambda \) is a constant and \( U \) is the constant free stream velocity or the velocity of the far field (inviscid) flow \([6]\). Coordinate system is considerate Cartesian \((x, y)\), where \( x \) and \( y \) are the coordinates measured along the plate and normal to it. When \( y \geq 0 \) flow takes place and temperature of moving plate is constant \( T_w \), ambient nanofluid temperature is \( T_e \) and we have heat transfer at \( T_w > T_e \). Using the nanofluid model proposed by Tiwari and Das \([3]\). The basic steady conservation of mass, momentum and energy equations are following:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]
The properties of nanofluid are defined as follows:

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}
\]

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s,
\]

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}},
\]

\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s,
\]

\[
k_{nf} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)},
\]

Here \(k_{nf}\) is thermal conductivity of nanofluid, \((\rho C_p)_{nf}\) is heat capacity of nanofluid, \(\phi\) is nanoparticle volume fraction, \(\rho_f\) is density of fluid, \(\rho_s\) is density of solid fraction, \(k_f\) and \(k_s\) are thermal conductivity of fluid and the solid fractions, \(\mu_{nf}\) is viscosity of fluid and \(\mu_f\) approximated by Brinkman [8]. Variables in boundary layer are follows:

\[
x = \frac{x}{L},\ y = \text{Re}^{1/2} \left(\frac{y}{L}\right),\ u = \frac{\bar{u}}{U}
\]

\[
v = \text{Re}^{1/2} \left(\frac{v}{L}\right),\ u_e = \frac{\bar{u}_e}{U}
\]

\[
\theta = \frac{T - T_w}{T_w - T_\infty}
\]

\[
P = \frac{\bar{P} - P_\infty}{(\rho_f U^2)},\ w = \frac{\bar{w}}{U}
\]

Here \(L\) is length of the plate, \(\text{Re}\) is Reynolds number.
In this problem we assumed have no pressure gradient so dimensionless boundary layer an equation follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (11)$$

$$\frac{u}{\partial x} + \nu \frac{\partial u}{\partial y} = \left( \frac{\mu_n}{\mu_f} \right) \frac{\partial^2 u}{\partial y^2}$$  \hspace{1cm} (12)$$

$$\frac{\partial \theta}{\partial x} + \nu \frac{\partial \theta}{\partial y} = \left( \frac{\alpha_n}{\nu_f} \right) \frac{\partial^2 \theta}{\partial y^2}$$  \hspace{1cm} (13)$$

For equations (5) boundary condition as follows:

$$u = u_w = \lambda, v = v_w, \theta = 1 \text{ at } y=0$$  \hspace{1cm} (14)$$

$$u = u_w = 1, \theta = 0, \text{ as } y \to \infty.$$

Note that for suction $\nu_w < 0$ and $v_w = 0$ for impermeable surface and for injection $\nu_w > 0$.

We can use similar solution for equations 11-13 as follows:

$$\eta = \frac{y}{(2x)^{1/2}},$$  \hspace{1cm} (15)$$

$$f(\eta) = \frac{\psi}{(2x)^{1/2}},$$  \hspace{1cm} (16)$$

$$g(\eta) = \theta,$$  \hspace{1cm} (17)$$

Note: that here $\psi$ is stream function and usually obtained from as this:

$$u = \frac{\partial \psi}{\partial y},$$  \hspace{1cm} (18)$$

$$v = -\frac{\partial \psi}{\partial x}.$$

For different order equations 11-13 we can use this formula:

$$v_w = -\frac{f_0}{(2x)^{1/2}},$$  \hspace{1cm} (19)$$

Here $f_0 > 0$ consider for suction, $f_0 < 0$ we have injection $f_0 = 0$ for impermeable surface and $f_0 = f(0)$ is non-dimensional constant for which determines the transpiration rate for them [9, 10] Using Eqs. (15)-(17) and (19), (12) and (13) are reduce to this ordinary differential equations:

$$\frac{f'''}{(1-\varphi)^{2.5}} \left[ 1 - \varphi + \varphi \left( \frac{\rho}{\rho_f} \right) \right] + ff'' = 0$$  \hspace{1cm} (20)$$

$$\frac{1}{Pr} \left[ 1 - \varphi + \varphi \left( \frac{\rho C_p}{\rho_f C_p} \right) \right] + fg' = 0$$  \hspace{1cm} (21)$$

Boundary conditions are as follows:

$$f(0) = f_0, f'(0) = \lambda, g(0) = 1$$

$$f'(\eta) = 1, g(\eta) = 0, \eta \to \infty.$$  \hspace{1cm} (22)
The physical properties are friction coefficient and the Nusselt number, which define as:

\[ C_f = \frac{\tau_w}{\rho_f u_w^2} \]

\[ Nu = \frac{L q_w}{k_f (T_w - T_\infty)} \]

Here \( \tau_w \) and \( q_w \) are surface shear stress and surface heat flux, which become to:

\[ \tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

From Eqs. (7) – (10) and (15)-(16), formulation (24) obtained:

\[ \left( 2 \text{Re}_x \right)^{1/2} C_f = \frac{f^*(0)}{(1-\varphi)^{2.5}}, \]

\[ \left( 2 / \text{Re}_x \right)^{1/2} Nu = -\frac{k_{nf}}{k_f} g^{''}(0) \]

III. RESULT AND DISCUSSION

For solve equations (20) and (21) use numerical solution and using the Runge-Kutta method with shooting technique. Result find missing slopes \( f^*(0), g^{''}(0) \) for different values parameters. Three types of nanoparticles were considered such as copper Cu, alumina \( Al_2O_3 \), \( TiO_2 \) (titanium) and parameters, namely the nanoparticle volume fraction \( \varphi \), the moving parameter \( \lambda \) and the suction or injection parameter \( f_0 \) considered [7]. For water the value of prandtl number pr as 6.2 and thermo physical properties of fluid and nanoparticles as follow in TABLE 1.

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Fluid phase (water)</th>
<th>Cu</th>
<th>Al2O3</th>
<th>TiO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kg K)</td>
<td>4179.000</td>
<td>385</td>
<td>765</td>
<td>686.2000</td>
</tr>
<tr>
<td>( \rho ) (Kg/m(^3))</td>
<td>997.100</td>
<td>8933</td>
<td>3970</td>
<td>4250.000</td>
</tr>
<tr>
<td>( K ) (W/mK)</td>
<td>0.613</td>
<td>400</td>
<td>40</td>
<td>8.9538</td>
</tr>
</tbody>
</table>

For regular Newtonian fluid \( \varphi = 0 \) and nanoparticles as 0 to 0.2 and \( f_0 \) as -0.6, 0 and 0.6. Figure 1 Nusselt number, eq. (26) with different values of \( f_0 \) and nanoparticle volume fraction parameter \( \varphi \) for three different values \( Cu, Al_2O_3 \) and \( TiO_2 \). This figure shows that, with increased volume fraction parameter these quantities increase.
Fig.1 Variation of the local Nusselt number with $\varphi$ for different nanoparticles and $f_0$, $pr = 6.2, \lambda = 0.5$

It is seen that, $TiO_2$ has the lowest value of thermal conductivity and lowest heat transfer rate.

Fig.2 Velocity profiles for Cu-water nanofluid and different values of $f_0$ and $\lambda = -1, pr = 6.2$.

Fig.3 Temperature profiles for Cu-water nanofluid and different values of $f_0$ and $pr = 6.2, \lambda = -0.1$.

Figure 2 and 3 show the velocity profile for Cu-water nanofluid and different values of $f_0$ for Pr as 6.2 and $\lambda = -0.1$. 
Fig. 4 Velocity profiles for Cu-water nanofluid and different values of $f_0$ when $pr = 6.2$ and $\lambda = -0.1$.

IV. CONCLUSIONS

Boundary layer equations were solved numerically by the Rung Kutta method with shooting technique. Results for injection and suction parameters $f_0$ and nanoparticle volume fraction $\Phi$, local Nusselt number $-g'(0)$ and moving parameter $\lambda$. Variation of the local Nusselt number with $\Phi$ for different nanoparticles and variable parameters is studied. It is shown that, $TiO_2$ has the lowest value of thermal conductivity and lowest heat transfer rate.

REFERENCES